These are answers to the even numbered problems from the homework, so you can check your results.

Section 2.6:

- 4. This is only true if A and B are square matrices. (Theorem 2.6.11)
- 18. The inverse is $\begin{pmatrix} -5 & 3 \\ 2 & -1 \end{pmatrix}$. The solution is $\begin{pmatrix} 4 & -1 \end{pmatrix}$
- 38. Check by multiplication if the inverse you get is correct.

Section 3.1:

- 2. True. Write out a general upper triangular matrix $\begin{pmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{pmatrix}$ and check.
- 6. False. $\begin{pmatrix} 5 & 3 \\ 2 & 1 \end{pmatrix}$ is a counter example.
- 22. The determinant is $18\exp(x)$ which is always non zero.

Section 3.2:

- 2. True. Result in both cases is $c \times det$.
- 6. True. $det(A).det(B) = det(B).det(A)$. det is a number, so multiplication is commutative, although it's not for A and B.
- 36. Ans=15
- 38. Ans=$(9/5)^3$
- 46. $1 = det(Id) = det(AA^{-1}) = det(AA^T) = det(A).det(A^T) = det(A).det(A) = det(A)^2$
- 52. Check.

Section 4.2

- T/F 4. True. (Addition is commutative.)
- Problem 12. It is a vector space. Check the axioms of a vector space, def 4.2.1 For instance

  A5. 1 is the zero vector

  A9. $(uv)^r = u^r v^r$
- Problem 20. $\mathbf{R}^3$ is not a complex vector space. For instance, $i(1,1,1) = (i,i,i)$ is not in $\mathbf{R}^3$. So not closed under scalar multiplication.

Section 4.3

- T/F 4. False. (Strictly speaking). For instance $(1,2) \notin \mathbf{R^3}$ even though we might think of it as lying in the x-y plane.
- Problem 22. Null space= $\{0\}$. Det is non zero, so unique solution.

Section 4.4

- T/F 4. False. Not uniquely. $\{i, j, i+j\}$ is a spanning set for the plane. We may use this to write the vector $i + 2j = 1(i+j) + 1.j, 1.i + 2.j$ in two different ways.
- T/F 12. True. If there's a finite spanning set, look at the highest degree polynomial in it and consider a polynomial of higher degree. This wouldn't be expressible in terms of your finite spanning set.
- Problem 12. (1,0,-1,1), (0,1,1,-2)
- Problem 28. (a) A general vector $= a\cosh(x) + b\sinh(x)$

  (b) A general vector as in (a) can be written as $(a/2 + b/2)e^x + (a/2 - b/2)e^{-x}$.

Section 4.6

- T/F False. $dim[P_n] = n + 1$ because of the constant term in polynomials. $dim(\mathbf{R}^n) = n$
- T/F 8. True. $M_3(\mathbf{R})$ has dimension 9.
- Problem 6. The matrix formed with the vectors as columns has determinant $1 - 2k + k^2$ which has to be nonzero for the vectors to be a basis. This happens when $k \neq 1$
- Problem 28. The conditions that the rows sum to zero and columns sum to zero give 6 equations. In a 9 dimensional space of matrices, We expect a basis of three matrices. Guess: $\begin{pmatrix} 0 & 1 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$, $\begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$,$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{pmatrix}$. Check that these are independent. Extend this to a basis by giving six independent vectors which dont lie in the subspace, i.e 6 vectors whose rows and columns dont add to 0. So,
  $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$
  Check that the matrices above are independent and their span doesn't intersect the subspace. So you have 9 independent vectors in $M_3(R)$, which has to be a basis. (Don't expect a question like this on the quiz.)
- Problem 34. $S = \{a_1(2x^2 + x + 3) + a_2(x^2 + x - 1)$. The polynomials in the brackets span the subspace S. Extend by the polynomial x^2 which is not in the span. (check). Or you can use any other polynomial that's outside the span, and prove that it's outside the span.

Section 4.7

- T/F 4. True. Look at the box on pg.298
- Problem 38. If every vector has a unique representation, then in particular, the vector 0 does. $c_1v_1 + c_2v_2 \dots c_kv_k = 0$ has the solution 0. So it is the unique solution. So the vectors are independent.

Section 4.9

- T/F 2. False Rowspace $\subset R^9$
- T/F 8. $BX = 0 \Rightarrow ABX = 0$. i.e X is in the nullspace of A imples it is in the nullspace of AB. So nullspace of AB is bigger.
- Problem 6. Rank 2. The second row is parallel to the first. Take away the last row from the first gives a 0001 rows which is independent from the other one.
- Problem 18. Hint.

Section 5.1

- T/F 2. False $m \times n$.
- T/F 4. True. With respect to the standard bases.
- T/F 6. True.
- Problem 4. Check $T(y_1 + y_2) = T(y_1) + T(y_2)$, $T(cy) = cT(y)$. Works because differentiation is linear.

Section 5.5

- T/F 4. False. Look at the box 2 on pg 385.
- T/F 6. True. Theorem 5.5.9(b)

- Problem 8 (a) . $\left(\begin{smallmatrix} 2 & 0 \\ 0 & -3 \end{smallmatrix}\right)$
  (b)$T(e^{2x} - 3e^{-3x}) = 2e^{2x} - 9e^{-3x} = -9(e^{2x} + e^{3x}) - 11(-e^{2x})$
  $T(2e^{-3x}) = -6e^{-3x} = -6(e^{2x} + e^{3x}) - 6(-e^{2x}))$
  We can write the components of the T(basis B) in the basis C as columns to get the matrix $\left(\begin{smallmatrix} -9 & -6 \\ -11 & -6 \end{smallmatrix}\right)$
  Problem 20. $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{bmatrix}$

Section 5.6

- 28.
- 34. Expand $det(A - \lambda I)$ by the last row etc.
- 36. $Av = \lambda v$, Multiply by A^{-1}

Section 5.8

- T/F 2. $A = SDS^{-1}$ implies $A^{-1} = SD^{-1}S^{-1}$, if the eigen values of A are nonzero.
- Problem 22. Did this in class
- 24.
- 32.

Section 5.9

- T/F 2. False. It should be $(A^2)/2!$.

Section 6.1

- T/F 2. False. You need wronskian to be zero at just one point for the solutions to be dependent.
- T/F 6. True. Check.

Section 6.2

- T/F 2. False. Did in class. Doesn't work for non constant coefficient equations.
- T/F 6. True.
- Problem 40. (a) Real roots are negative. (b)Real parts of the equations are negative. (c) Use the fromula for the roots of the quadratic equation and the parts (a) and (b). (e) Solutions are sin and cos, which are bounded.

Section 6.3

- T/F 2. False D^k annihilates only up to x^{k-1}
- T/F 6. True. The homogenous equation after you apply the annihilator is $D^4(D^2 + 4) = 0$

Section 6.4

- T/F 2. True.
- Problem 4. Use trignometric identity $\sin^2(x) = (1-\cos(2x))/2$. Ans: $y_p = sin(2x)+3cos(2x)-10$

Section 6.5

- T/F 4. True. The $e^{-ct/2m}$ goes to zero as $t \to \infty$).
- Problem 26.

Section 6.9

- Problem 2. $c_1 x \cos(x) + c_2 x \sin(x)$.
- problem 8. $4a_2 = a_0$, $a_1 = 0$, So $y_1(x) = (4 + x^2)$ for instance. General solution $c_1(x^2 + 4)tan^{-1}(x/2) + c_2(x^2 + 4)$ **This is probably wrong. Did in class**

Section 7.1

- T/F Review 8.True
- Problem 18. Set $y = x_1, y' = x_2, y'' = x_3$. Then,

$$
\begin{aligned}
x_1' &= x_2 \\
x_2' &= x_3 \\
x_3' &= e^t x_1 - t^2 x_2 + t
\end{aligned}
$$

- 20

Section 7.2

- T/F Review 2. False $W(x_1, x2) = -W(x_2, x_1)$
- T/F Review 6. True
- Problem 12. $c_1(4, -1)e^t + c_2(1, 1)e^{-2t}$

Section 7.3

- Problem 2. Check.

Section 7.4

- T/F Review 2. True. (Thm 7.4.8)

- T/F Review 4. False. Only if A is diagonalisable.