Notes

In these notes, "page x (y)" means page x in the original publication and page y in this volume; "page x (vol. I, p. y)" means page x in the original and page y in [SP1].

[61a] **The topology of normal singularities of an algebraic surface and a criterion for simplicity.**

1. Page 5 (3), line -2 (a note concerning the Theorem): The generalization of the Theorem for (V^n, P) becomes false for every $n > 2$. Brieskorn has produced examples in which P is a normal isolated hypersurface singularity of type $z_0^{a_0} + \ldots + z_n^{a_n} = 0$. See *Proc. Nat. Acad. Sc. USA* **55** (1966) 1395–1397, and *Invent. Math.* **2** (1966) 1–14.
2. Page 6 (4), line 3 of the 4th paragraph:

   "note (a) $S_{ij} \geq 0$" should read "note (a) $S'_{ij} \geq 0$".

   Of course these two statements are equivalent, but next sentence refers to properties of the matrix S'.
3. Page 15 (13), line -5: "minimal prime ideal" should read "height one prime ideal".
4. Page 16 (14), lines -5 and -4: The first conjecture in the last paragraph on this page, that the ideal class groups of the holomorphic local ring $\mathfrak{o}$ and its completion $\mathfrak{o}^*$ are canonically isomorphic, is true. In fact the obvious generalization to a henselian pair (A, I) is also true; see the Corollary at the bottom of p. 573 of R. Elkik, Solutions d'équations à coefficients dans un anneau hensélien, *Ann. scient. Éc. Norm. Sup.*, Ser. 4, **6** (1973) 553–603.
   Notice that the holomorphic local ring $\mathfrak{o}$ is strictly henselian. The case where $A = \mathfrak{o}$ is a two-dimensional henselian local ring was proved by Boutot in 1971.[1]

[1] J.-F. Boutot, Groupe de Picard local d'un anneau hensélien, *C. R. Acad. Sc. Paris* **272** (1971) A1248–A1250.

5. Page 16 (14), lines -3 and -2: A modified version of the second conjecture in the last paragraph is true. Denote by $\hat{X}$ the formal completion of the reduced exceptional divisor $E := E_1 + \cdots + E_n$ in F'. Then there exists an exceptional divisor $Z \geq E$ supported in E such that the natural map $\mathrm{Pic}(\hat{X}) \to \mathrm{Pic}(D)$ is an isomorphism for every divisor $D \geq Z$ supported in E. This follows from Lemma 2.10 on p. 494 of M. Artin, Some numerical criteria for contractibility of curves on an algebraic surface, *Amer. J. Math.* **84** (1962) 485–496, and the standard GFGA theorems. The (strict interpretation of the) original conjecture, that the natural map $\mathrm{Pic}(\hat{X}) \to \mathrm{Pic}(E)$ is an isomorphism, is false. An example is provided by Example 6.3 (i) on p. 193 of H.C. Pinkham, Normal surface singularities with $\mathbb{C}^*$ action, *Math. Ann.* **227** (1977) 183–193. In this example, $E = E_1 + E_2 + E_3$, where E_1 is a smooth elliptic curve, E_2 and E_3 are isomorphic to $\mathbb{P}^1$, with the following intersection numbers: $(E_1 \cdot E_1) = -1$, $(E_2 \cdot E_2) = (E_3 \cdot E_3) = -2$, $(E_1 \cdot E_2) = (E_2 \cdot E_3) = 1$, $(E_1 \cdot E_3) = 0$. Moreover the restriction $\mathrm{Tr}_{E_1}(E) := \mathscr{O}(E) \otimes \mathscr{O}_{E_1}$ of $\mathscr{O}(E)$ to the elliptic curve E_1 is trivial. The short exact sequence $0 \to \mathrm{H}^1(E_1, \mathrm{Tr}_{E_1}(-E)) \to \mathrm{Pic}(E + E_1) \to \mathrm{Pic}(E) \to 0$ shows that the strict version of the second conjecture fails.

[61b] **Pathologies of modular algebraic surfaces.**

1. The displayed formula on p. 342 (vol. I, p. 734) should read

$$\phi_1^*(dt/t) = \phi_2^*(dt/t) = \phi_3^*(dt/t) = d(xyz)/(xyz)\,.$$

[62b] **The canonical ring of an algebraic surface.**

1. The canonical ring of a complex algebraic variety of general type has since been proved to be finitely generated in any dimension. An analytic proof was given by Y.-T. Siu, Finite generation of canonical ring by analytic method, *Sci. China* Ser. A 51 (2008) 481–502. An algebraic proof exists in preprint form in C. Birkar, P. Cascini, C. Hacon and J. McKernan, *Existence of minimal models for varieties of log general type.*
2. Page 613 (22): In the first displayed formula, "$(K^2) - \deg(c_2)$" should read

$$(K^2) + \deg(c_2)\,.$$

3. Page 614 (23), footnote: The cited paper by M. Artin appeared in *Amer. J. Math.* **84** (1962) 485–496.

[65a] **A remark on Mordell's conjecture.**

1. Page 1008 (47), line 15: "V_k" should be "X_k".

2. Page 1009 (48), line -2: In the definition of a *divisorial correspondence*, instead of requiring $\delta \in \mathrm{Pic}(X \times Y)$ to be trivial when restricted to both $X \times \{p_Y\}$ and $\{p_X\} \times Y$, one should impose the first (resp. the second) condition only when $\dim(Y) > 0$ (resp. $\dim(X) > 0$). (The universal mapping property (*) on p. 1010 (49) then gives a k-rational point $\tilde{\eta} \in J_k$ for any element $\eta \in \mathrm{Pic}(C)$ of degree 0 in the Picard group of C as in the proof of Proposition 2 on p. 1013 (52).)
3. Page 1010 (49), line 17: "J is characterized" should read "$\hat{J}$ is characterized".
4. Page 1010 (49), line -4: "with $X = J$" should read "with $X = \hat{J}$".
5. Page 1014 (53), line 7: "$p_1^*(x\ )$" should read "$p_1^*(x_0)$".

[65b] **Picard groups of moduli problems.**

1. Page 33 (56), line 8: There has been enormous progress on the question of the rationality of M_g, which remains a subject of active research. In characteristic 0 the following are some highlights.
   - $\mathscr{M}_g$ is of general type if $g \geq 24$. See [83]; J. Harris, Invent. Math. 75 (1984) 437–466; J. Harris and D. Eisenbud, Invent. Math. 90 (1987) 359–387.
   - $\mathscr{M}_g$ is rational for $g = 1, 2, 4, 6$. See I. Dolgachev, Proc. Symp. Pure Math. 46, Part 2, 1987, pp. 3–16 for more information.
   - $\mathscr{M}_g$ is unirational for $g \leq 13$. For $g \leq 10$ this is classical and was known to Severi in 1915. For $g = 12$ see E. Sernesi, *Ann. Sc. Ec. Norm. Sup. Pisa* **8** (1981) 405–439; for $g = 11$ or 13 see M.C. Chang and Z. Ran, *Invent. Math.* **76** (1984) 41–54.
2. Page 47 (70), 3rd line of §3: Insert "complete" after "reduced".
3. Page 49 (72), in the displayed formula (b): The term on the far right, i.e., the expression "$[(S_2 \times \mathscr{X}_2) \times_{(S_1 \times S_2)} T]$", should read
$$[(S_1 \times \mathscr{X}_2) \times_{(S_1 \times S_2)} T].$$
4. Page 53 (76), line 15: In the displayed formula of line -14, that is right after the phrase "We get a diagram", add g below the arrow $T \longrightarrow S$, to become $T \underset{g}{\longrightarrow} S$.
5. Page 57 (80), line 14: "$\frac{\lambda-1}{\gamma}$" should read "$\frac{\lambda-1}{\lambda}$".

[67d] **Abelian quotients of the Teichmüller modular group.**

1. Page 227 (105): See the extensive comments on a preprint version of this paper by Grothendieck in his 1966 May 9 letter to Mumford (p. 717 in this volume).
2. Page 228 (106), line -4: It is clear from the context that the precise conjecture here is that the rank of $\mathrm{H}^2(\Gamma_g, \mathbb{Z})$ is one for $g \geq 3$: as mentioned on p. 243 (121), $H^2(\Gamma_2, \mathbb{Z}) \cong \mathbb{Z}/10\mathbb{Z}$. The first significant breakthrough on this question was

made by J. Harer, who showed in 1983 that $\mathrm{H}^2(\Gamma_g,\mathbb{Z}) \cong \mathbb{Z}$ for $g \geq 5$; see *Invent. Math.* **72** (1983) 221–239. Note that the claim in the paper that the torsion part of $\mathrm{H}^2(\Gamma_g,\mathbb{Z})$ is isomorphic to $\mathbb{Z}/(2g-2)\mathbb{Z}$ was incorrect. Since then there has been enormous progress on the cohomology of the mapping class group and the cohomology of $\mathscr{M}_{g,n}$ and $\overline{\mathscr{M}}_{g,n}$. In particular, $\mathrm{H}^2(\Gamma_g,\mathbb{Z})$ has been computed: it is isomorphic to $\mathbb{Z}$ for $g > 2$, and is isomorphic to $\mathbb{Z}/10\mathbb{Z}$ for $g = 2$.

3. Page 233 (111), line -3: replace "$x - x^{\gamma}$" by "$x - x^{y}$"
4. Page 244 (122): Paper [8] in Bibliography is [u64b].

[69a] **Enriques' classification of surfaces in char p.**

1. Page 329 (vol. I, p. 664), lines 6, 9, 10, 11: "$\mathscr{O}_{E_i}(-E_i^2)$" should read "$\mathscr{O}_{E_i}(-E_i)$"; similarly in line 8, "$H^1(\mathscr{O}_{E_i}(-E_i^2))$" should read "$H^1(\mathscr{O}_{E_i}(-E_i))$"

[69b] **Biextension of formal groups.**

1. Page 307 (141), lines 2 and 3: In a letter to P. Cartier in the winter of 1967/8, Mumford explained the definition of a multiplicative biextension of commutative formal groups and a related notion of a "polarized canonical module" over a commutative ring R of characteristic p and outlined a proof that generic abelian varieties in characteristic p are ordinary, in (a) and (b) below, similar to (ii)–(iv) on p. 307: (a) when R is a complete noetherian local ring, a polarized canonical module ovr R determines a deformation of polarized p-divisible groups over R; (b) a "generic" polarized canonical module deforming that of an abelian variety X over $R/\mathfrak{m}_R$ can be shown to be ordinary, by reducing to the case when the original abelian variety X satisfies $\alpha(X) = 1$.

   A version of the proof sketched in steps (i)—(iv) was published by P. Norman and F. Oort, *Ann. Math.* **112** (1980) 413–439. The proof uses a version of the method mentioned in (iii) below, published by P. Norman in *Ann. Math.* **101** (1975) 499–509. The notion of a *displayed Dieudonné module* in Norman's paper was generalized by T. Zink into a new Dieudonné theory for p-divisible groups, called the *theory of displays*. See T. Zink, The display of a formal p-divisible group, in *Cohomologies p-adiques et applications arithmétiques I*, Astérisque 278 (2002), 127–248; T. Zink, A Dieudonné theory for p-divisible groups, in *Class field theory—its centenary and prospect (Tokyo, 1998)*, Adv. Stud. Pure Math. 30, Math. Soc. Japan, Tokyo, 2001, 139–160; W. Messing, Travaux de Zink, *Séminaire Bourbaki* 2005/2006, Exp. 964, Astérisque 311 (2007), ix, 341–364.
2. Page 307 (141), line -6: Cartier's results were published by M. Lazard in LNM 443, Springer-Verlag, 1975.
3. Page 308 (142), line 5: The following should be added as a footnote for the definition of the ring A_R:

'The ring A_R is the completion of the noncommutative ring $W(R)[V][F]$ with respect to the right ideals $V^n W(R)[V][F]$. Explicitly, the ring A_R consists of all infinite series of the form

$$\sum_{m,n\geq 0} V^m[a_{mn}]F^n \qquad \text{with } a_{mn}\in R\ \forall\ m,n\geq 0$$

with the property that for every $m \geq 0$, there exists an integer C_m such that $a_{mn} = 0$ for all $n \geq C_m$. Here $[a_{mn}]$ denotes the Witt vector $(a_{mn},0,0,\ldots)$, the "Teichmüller representative" of a_{mn}.'

4. Page 310 (144), §2: The notion of biextension was further developed in Exposés VII, VIII of SGA7I, LNM 288, Springer-Verlag, 1972. The set of isomorphism classes of biextensions of $G\times H$ by F is $\mathrm{Ext}^1(G\otimes^{\mathbb{L}} H,F)$, when G,H,F are abelian groups in a topos. In LNM 980, L. Breen developed the notion of *cubical structures* and related it to biextensions.
5. Page 313 (147), line -10: "Φ,Ψ are the group laws of G and H respectively" should read "Φ,Ψ are the group laws of H and G respectively".
6. Page 319 (153), line 3: "$\beta(Pm,Qn) = P.(m,n).Q^*$" should read "$\beta(Pm,Qn) = P\cdot\beta(m,n)\cdot Q^*$".

[69d] **Rational equivalence of 0-cycles on surfaces.**

1. Page 197 (vol. I, p. 755), 3 lines above the displayed commutative square: "induced 2-form" should read "induced q-form".

[70] **Varieties defined by quadratic equations.**

1. Page 70 (232), line 4 of the second paragraph: The word "natural" should be eliminated. In fact the homomorphism $\rho_\alpha : K(L) \to \mathscr{G}(P_\alpha)$ which splits the extension
$$1\to k^*\to \mathscr{G}(P_\alpha)\to K(L)\to 1$$
is unique up to $\mathrm{Hom}(K(L),k^*)$.
2. Page 75 (237), line 12: W denotes the image of
$$\sum_{\alpha\in\hat{X}} \Gamma(L\otimes P_\alpha)\otimes\Gamma(M\otimes P_{-\alpha})\to\Gamma(L\otimes M)$$
as in the proof of the Lemma on p. 68 (230).
3. Page 79 (241), line 2 from bottom: The displayed formula should read
$$P(x,y) = \chi(L)\prod_{i=1}^{g}(x-\alpha_i y)\,.$$

4. Page 80 (242): The displayed formula on line 8 should read

$$P(x,y) = (\text{constant}) \cdot \prod_{i=1}^{r} (x - \alpha_i y) \cdot y^{g-r}.$$

5. Page 99 (261), line 4: Insert “abelian” between “complementary” and “subvariety”.

[71a] Theta characteristics of an algebraic curve.

1. Page 189 (vol. I, p. 488), line 12: “E is a $\pi^*\mathscr{O}_{X'}$-algebra” should read “E is a $\pi_*\mathscr{O}_{X'}$-module”.

[71b] A remark on Mahler’s compactness theorem.

1. Page 291 (265): The last displayed formula (second formula in the statement of Theorem 2) should read

$$\left\{ \Gamma \in \mathfrak{M}_G^C \mid \Gamma \cap U_\varepsilon = \{e\},\ \text{measure}(G/\Gamma) \leq D \right\}$$

(in other words, the superscript “C” should be moved from Γ^C to $\mathfrak{M}_G^C$).

[72d] Introduction to the theory of moduli.

1. Page 176 (274), 4 lines before Corollary: “$\psi \colon M \to N$” should read “$\Psi \colon M \to N$”.

[73c] A remark on the paper of M. Schlessinger.

1. Page 117 (326), the last line: The paper by H. Pinkham was published in *J. Algebra* **30** (1974) 92–102.

[75b] Matsusaka’s big theorem.

1. Page 513 (327): A refinement of Theorem 1 by J. Kollár and T. Matsusaka states that k_0 depends only on the first two coefficients of the Hilbert polynomial $P(k)$; see *Amer. J. Math.* **105** (1983) 229–252. An effective estimate of k_0 is provided in Y.-T. Siu, *Ann. Inst. Fourier* **43** (1993) 1387–1405. An exposition of the effective version of Matsusaka’s big theorem is given in 10.2 of the book *Positivity in Algebraic Geometry. II: Positivity for Vector Bundles and Multiplier Ideals* by R. Lazarsfeld, Springer-Verlag, 2004.
2. Page 529 (343), line 7: Insert “by” between “parametrized” and “a suitable countably infinite set”.

[76a] Hilbert's fourteenth problem—the finite generation of subrings such as rings of invariants.

1. Page 438 (359): In the displayed formula in line -8, "$1, f_1, \ldots, f_{k-1}$" should read

$$1, \mathscr{L}(bH + aE), \mathscr{L}(2bH + 2aH), \ldots, \mathscr{L}((k-1)bH + (k-1)aE).$$

2. Page 443 (364): The paper by E. Formanek and C. Procesi mentioned in Added in proof was published in *Advances in Math.* **19** (1976) 292–305.
3. Page 444 (365): Entry [25] of REFERENCES: W.J. Haboush, *Reductive groups are geometrically reductive*, was published in *Ann. Math.* **102** (1975) 67–83.

[76b] The projectivity of the moduli space of stable curves I: Preliminaries on "det" and "Div".

1. Page 20 (367), line -14: This display formula should be

$$\psi(l \otimes m) = (-1)^{\alpha(x)\beta(x)}(m \otimes l).$$

[78a] An algebro-geometric construction of commuting operators and of solutions to the Toda lattice equation, Korteweg de Vries equation and related nonlinear equations.

1. Page 116 (404), line 12: "$[a_1, a_2]$, $[b_1, b_2]$" should be "$[-a_1, a_2]$, $[-b_1, b_2]$". In §1, it is also assumed that a_1, a_2, b_1, $b_2 > 0$.
2. Page 116 (404), line -9: "$T_{x,p}$" should be "$T_{X,P}$".
3. Page 119 (407), 1st line of Proposition: "$\{X, P, Q, R\}$" should be "$(X, P, Q, \mathscr{F})$".
4. Page 120 (408), lines 5–8: "$|x_{n+1}| \geq C|x_n|$" (line 8) should read "$|x_{n-1}| \geq C|x_n|$". For conditions "$|x_n| \geq C|x_{n-1}|$" or "$|x_{n-1}| \geq C|x_n|$" to define neighborhoods of the two points at infinity, we should assume $\Sigma \cap \Theta = \emptyset$ as in p. 127 (415), and take a suitably normalized R among the equivalent rings, e.g., one for which A satisfies $A_{i,i+a_2} = 1$.
5. Page 120 (408), line -8: "$BC = CA$" should be "$BC = CB$".
6. Page 121 (409), line -13 [4th displayed formula]: "$\dfrac{a_2 - 1}{a_1} \geq \dfrac{\lambda}{\mu} \geq \dfrac{b_2}{b_1 - 1}$" should be "$\dfrac{a_2}{a_1} > \dfrac{\lambda}{\mu} > \dfrac{b_2}{b_1}$" as in the first displayed formula in the page, and subsequent arguments in p. 122 (410) should be changed appropriately, e.g., in line 5: "$R_{\lambda(a_2+b_1-1)} \cap R^{\mu(a_2+b_1-1)}$" should read "$R_{\lambda(a_2+b_1)-1} \cap R^{\mu(a_2+b_1)-1}$", etc. (Otherwise we need to also assume $(a_2 - 1)/a_1 \geq b_2/(b_1 - 1)$; this can be achieved, e.g., by replacing A and B by suitable powers of them.)
7. Page 121 (409), line -7: "$A^{\lambda} \in \mathscr{R}_{a_1}$" should read "$A^{\lambda} \in \mathscr{R}_{a_2}$".

8. Page 127 (415), line 6: "$(n,a_1)=(n,a_2)=0$" should be "$(n,a_1)=(n,a_2)=1$". This may be regarded as a special case of the condition $(a_1,b_1)=(a_2,b_2)=1$ in Data B. When we consider $[A,S^n]=0$ without the restriction on (n,a_1) and (n,a_2), the resulting picture looks simpler than the general Data A′–Data B′ correspondence: see [79b].
9. Page 133 (421), line 2 of Proof of Lemma: "$\deg A=\alpha$" should be "$\deg A=a$", and "$a+n+1$" should be "$a+n-1$".
10. Page 136 (424), line -6 and Page 137 (425), line 1: $\bigoplus_{i=1}^{r-1}$ should be $\bigoplus_{i=0}^{r-1}$.
11. Page 153 (441), line 13: H. McKean's article appeared in *Partial differential equations and geometry (Proc. Conf., Park City, Utah, 1977)*, Lecture Notes in Pure and Appl. Math., 48, Dekker, New York, 1979, pp. 237–254.
12. Page 153 (441), line 16: Airault *et al*.'s article appeared in *Comm. Pure Appl. Math.* **30** (1977), no. 1, 95–148. For further development in this direction, see I. Krichever, Elliptic solutions of Kadomtsev-Petviashvili equations and integrable systems of particles, *Funct. Anal. Appl.*, **14** (1980), no. 1, 45–54 (In Russian), 282–290 (Translation), A. Treibich and J.-L. Verdier, Solitons elliptique, in *The Grothendieck Festschrift*, Vol. III, Progr. Math. 88, Birkhäuser, 1990, pp. 437–480, and the literature cited therein.

[78c] **Some footnotes to the work of C.P. Ramanujam.**

1. Page 249 (447), line 3 of the footnote: "y^t" should read "Y^t".
2. Page 250 (448), bottom of the page: The displayed statement (1) is part of the Theorem on page 121 of C.P. Ramanujam, Supplement to the article "Remarks on the Kodaira vanishing theorem", *J. Indian Math. Soc.* **38** (1974), 121–124.
3. Page 251 (449), line 8: The paper by D. Gieseker is On a theorem of Bogomolov on Chern classes of stable bundles, *Amer. J. Math.* **101** (1979) 77–85.
4. Page 251 (449): Mumford's proof of Kodaira vanishing theorem (the proof (2)$\Longrightarrow$(1), starting on line 9) also appeared in Reid's article [13] as an appendix.
5. Page 254 (452), line 5: The morphism "resp" is $p|_E$, the restriction to E of the morphism $p: F\to C$.
6. Page 254 (452): The displayed formula before "*Q.E.D.*" should read
$$\mathrm{H}^1(F_0,\mathscr{O}(-H))\neq(0).$$
7. Page 255 (453), line 3 of Theorem: "$\pi\colon X^*\to \operatorname{Spec}\mathscr{O}$" should read "$\pi\colon X\to \operatorname{Spec}\widetilde{\mathscr{O}}$".
8. Page 257 (455), line -4: The result of H. Matsumura referred to is in the article Geometric structure of the cohomology rings in abstract algebraic geometry, *Mem. Coll. Sci. Univ. Kyoto*, Ser. A Math. **32** (1959) 33–84.
9. Page 258 (456): The beginning of the two displayed lines in the statement of the Lemma should be "$H^i_{\{x\},\mathrm{int}}(\mathscr{O})$".

10. Page 258 (456): The local cohomology groups in the displayed lines −8 and −6 should be $H^i_{\pi^{-1}x}(\mathscr{F})$ and $H^i_{\pi^{-1}x}(\mathscr{O}_X)$, respectively.
    Line −2: "$R_i\pi_*(\Omega^n_X)$" should read "$R^i\pi_*(\Omega^n_X)$".
11. Page 261 (459), line 10: The referenced Corollary of Proposition 2.6 in Chapter V of article [17] appeared on page 144 of LNM 632, Springer-Verlag, 1978.
12. Article [2] by F. Bogomolov appeared as Holomorphic tensors and vector bundles on projective manifolds, *Izv. Akad. Nauk SSSR* Ser. Mat. **42** (1978) 1227–1287, 1439.
13. Article [3] by D. Buchsbaum and D. Eisenbud appeared in *Amer. J. Math.* **99** (1977) 447–485.
14. Article [13] by M. Reid appeared in *Proceedings of the International Symposium on Algebraic Geometry (Kyoto, 1977)*, Kinokuniya Book Store, Tokyo, 1978, pp. 623–642.
15. Article [17] by J.-F. Boutot appeared as LNM 632, Springer-Verlag, 1978.

[82] **On the Kodaira dimension of the moduli space of curves.**

1. A gap in the proof of Theorem 4 on p. 58 (vol. I, p. 206) was found by S. Mochizuki; see the Remarks on p. 372 and p. 392 of The geometry of the compactification of the Hurwitz scheme, *Publ. RIMS Kyoto Univ.* **31** (1995) 355–441. In §3 *loc. cit.*, one finds an exposition of the notion of *log admissible covering* and a proof of the existence of an algebraic stack with a log structure which classifies log admissible coverings; see Thm. 3.22 on p. 389 *loc. cit.* This supplies a proof of Theorem 4 on p. 58 (vol. I, p. 206); see §3.1 and the Remark on p. 377 *loc. cit.*