

# Math 644, Problem set 5 due November 20, 2007

Dr. Epstein

**Reading:** Taylor sections 3.6–3.11

1. If  $u$  is a homogeneous distribution of order  $j$ , and  $\alpha$  is a multi-index, show that  $\partial_x^\alpha u$  is a homogeneous distribution of order  $j - |\alpha|$ .

2. Prove that, as elements of  $\mathcal{S}'(\mathbb{R})$ ,  $\partial_x \log|x| = \text{P.V.}(x^{-1})$ . The locally integrable function

$$u(x) = \chi_{(0,\infty)}(x) \log x - \chi_{(-\infty,0)}(x) \log(-x).$$

defines another distribution. Is the distribution  $\partial_x u$  homogeneous of degree  $-1$ ? Does it agree with the finite part distribution  $\text{PF}(|x|^{-1})$ ? Why or why not?

3. Taylor page 225, problem 4.

4. Taylor page 238, problem 1.

5. Taylor page 238, problem 3, 4. You may want to look at section 3.1, and may use results from that section.

6. Taylor page 245, problem 5. This can be done for the case of  $\mathbb{R}^1$ .

7. Taylor page 248, problem 2.