Math 103: Secants, Tangents and Derivatives

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Outline

1. Review
2. Secant Lines and Tangent Lines
3. Derivative

Limits Involving Infinity

1. Limits as x approaches ∞ or $-\infty$.
2. Limits at infinite discontinuities
3. Horizontal asymptotes
4. Vertical asymptotes

Horizontal asymptotes

Definition

A line $y = b$ is a horizontal asymptote of the graph of $y = f(x)$ if

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Example: Find the horizontal asymptotes of $y = \frac{x+1+sin(x)}{x}$

Secant lines

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Find the secant line to $y = x^3 - 2x + 1$ between $x = 0$ and $x = 1$.

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"kisses" $=$ (near the point of intersection the graph is completely contained to one side of the tangent line)

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Tangent lines are the limits of secant lines

Tangent Lines

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The tangent line to a curve $y = f(x)$ at a point $(a, f(a))$ is the line through $(a, f(a))$ with the slope

$$lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

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Find the slope of the line tangent to $y = sin(x)$ at $x = 0$.

Application: Instantaneous Velocity

Definition

If $s(t)$ is a position function defined in terms of time t, then the instantaneous velocity at time $t = a$ is given by

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ExampleSuppose a penny is dropped from the top of DRL which is 19.6 meters high. The position of the penny in terms of hight above the street is given by $s(t) = 19.6 - 4.9t^2$. At what speed is the penny traveling when it hits the ground?

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Notation. Other ways of writing the derivative of $y = f(x)$.

$$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx}f(x) = Df(x) = D_x f(x)$$

The Sandwich Theorem

Theorem

If $f(x) \leq g(x) \leq h(x)$ *when* x *is near* c *and*

$$lim_{x \to c} f(x) = lim_{x \to c} h(x) = L$$

$$then\ lim_{x \to c} g(x) = L$$