# Math 103: Concavity and Using Derivatives to Graph a Function

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# Outline

## First Derivative Test

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2. If $f'$ changes from negative to positive at $c$, then $f$ has a local minimum at $c$.
3. If $f$ does not change sign at $c$, then $f$ has no local maximum or minimum at $c$.

## Definition

If a graph of $f$ lies above all of its tangents on an interval $I$, then is is called **concave up** on $I$. If a graph of $f$ lies below all of its tangents on an interval $I$, then is is called **concave down** on $I$.

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## Concavity test

1. If $f''(x) > 0$ for all $x$ in $I$, then the graph of $f$ is concave up on $I$.
2. If $f''(x) < 0$ for all $x$ in $I$, then the graph of $f$ is concave down on $I$.

**Definition**

A point $P$ on a continuous curve $y = f(x)$ is called an **inflection point** if $f$ changes from concave down to concave up or visa versa at $P$.

## The Second Derivative Test

Suppose $f''$ is continuous near $c$.

1. If $f'(c) = 0$ and $f''(c) > 0$, then $f$ has a local minimum at $c$.
2. If $f'(c) = 0$ and $f''(c) < 0$, then $f$ has a local maximum at $c$.

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6. Identify any asymptotes.
7. Plot key points and asymptotes, and sketch the curve.

**Definition**

The line $y = mx + b$ is a slant asymptote for $f(x)$ if

$$lim_{x\to\infty}[f(x) - (mx + b)] = 0$$

If $f(x) = \frac{p(x)}{q(x)}$ where $q(x)$ and $p(x)$ are polynomials, then $f(x)$ has a **slant asymptote** if and only if the degree of $p(x)$ is one more than the degree of $q(x)$.