Math 103: L'Hopital's Rule

Ron Donagi

University of Pennsylvania

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Outline

Indeterminate forms

For some limits evaluation via substitution gives meaningless expressions called **Indeterminate Forms**

$$lim_{x\to 0}\frac{sin(x)}{x} = \frac{0}{0}$$

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Other indeterminate forms include $\infty \cdot 0$, 0^0 and 1^∞

L'Hopital's Rule for $\frac{0}{0}$

Theorem

Suppose $f(a) = g(a) = 0$, f and g are differentiable near a and $g'(x) \neq 0$ for x near a but not equal to a, Then

$$lim_{x\to a}\frac{f(x)}{g(x)} = lim_{x\to a}\frac{f'(x)}{g'(x)}$$

if the right-hand limit exists.

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The theorem also holds for one-sided limits and infinite limits.

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This only helps us with indeterminate forms $\frac{0}{0}$.

L'Hopital's Rule for $\frac{\infty}{\infty}$

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Suppose $f(x) \to \infty$ and $g(x) \to \infty$ as $x \to a$, then

$$lim_{x \to a} \frac{f(x)}{g(x)} = lim_{x \to a} \frac{f'(x)}{g'(x)}$$

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if the right-hand limit exists.

The theorem also holds for one-sided limits and infinite limits.

Must convert other indeterminate forms to $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

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5. Find the points of inflection and the concavity of f.
6. Identify any asymptotes.
7. Plot key points and asymptotes, and sketch the curve.