Math 116 Sample Exam #2 November, 2008

Instructions: This exam consists of five problems. Do all five, showing your work and explaining your assertions. Give yourself 50 minutes. Each problem is worth 20 points.

1. Let $v, w, z \in \mathbb{R}^3$.

a) Show that if $v + w + z = 0$ then the set $\{v, w, z\}$ cannot span $\mathbb{R}^3$.

b) Show that if $v \times w = v - w$, then $v = w$.

2. a) Find a vector $z \in \mathbb{R}^3$ of length 1 that is orthogonal to the vectors $v = (1, 1, 0)$ and $w = (0, 1, 1)$.

b) With v, w, z as in part (a), find the volume of the parallelepiped generated by these three vectors.

3. a) Find the focus and directrix of the parabola $y = 2x^2$.

b) Find all points of this parabola where the curvature is maximal. Is there any point at which the curvature is minimal?

4. Let $P = (1, 1, 1)$ and $Q = (2, 3, -1)$ in $\mathbb{R}^3$. Let L be the line in $\mathbb{R}^3$ that passes though P and Q.

a) Find a differentiable function $F : \mathbb{R} \to \mathbb{R}^3$ that parametrizes the line L, such that $F(0) = P$, and such that the speed at every point is 1. Find the acceleration at each point.

b) Find a plane in $\mathbb{R}^3$ that is perpendicular to L and passes through the point P.

5. Let $F : \mathbb{R} \to \mathbb{R}^2$ be a differentiable function. For each of the following assertions, either give a proof or a counterexample.

a) If F parametrizes a circle centered at the origin, then the acceleration vector at each point is a multiple of the position vector.

b) If the velocity is constant, then the curvature is 0.