Math 116 | Problem Set #4 | Due Fri., Oct. 9, 2009, in class.

Reminder: The first exam is on Monday, October 5 in class. One two-sided handwritten 5"x 7" index card is permitted during the exam, but no other aids.

Read Apostol, Chapter 4, sections 3-5, 10, 13, 14, 16, 17. Optional: In Chapter 4 also read sections 1, 2, 7, 8, 11, 18, 20.

1. From Apostol, section 4.6, pages 167-168, do problems 12 and 38; and in section 4.9, page 173, do problems 1, 8, 11.

2. From Apostol, section 4.12, page 179-180, do problems 4 and 16; and in section 4.15, page 186, do problems 1 and 4.

3. Let $f(x) = 1$ if the integer $[x]$ is even, and let $f(x) = -1$ if $[x]$ is odd. Let $F(x) = \int_0^x f$ and let $\Phi(x) = \int_0^x F$. Graph the functions f, F, Φ. Are these functions integrable? continuous? differentiable?

4. a) Determine whether the function $f(x) = x^3 - x + 1$ has a maximum value and whether it has a minimum value on the closed interval $[-1, 2]$. If such values exist, find them and find for which values of x they are achieved. Relate your answer to the Extreme Value Theorem.
   b) Redo part (a) on the open interval $(-1, 2)$.

5. Which of the following functions are differentiable at $x = 0$? For each one that is, find $f'(0)$, and determine whether the function f' is continuous at $x = 0$.
   a) $f(x) = \sin(1/x)$ for $x \neq 0$, $f(0) = 0$.
   b) $f(x) = x \sin(1/x)$ for $x \neq 0$, $f(0) = 0$.
   c) $f(x) = x^2 \sin(1/x)$ for $x \neq 0$, $f(0) = 0$.
   d) $f(x) = x^3 \sin(1/x)$ for $x \neq 0$, $f(0) = 0$.

6. Let f be the function given in problem 6 of Problem Set 2. For which real numbers a in $[0, 1]$ is the function f differentiable at $x = a$? Prove your assertion.