

Read Apostol, Chapter 4, section 22; and Chapter 12, sections 1-10.

1. From Apostol, 4.23, page 201, do problems 1, 2, 10. In #1 and 2, also find all critical points (i.e. points where  $f_x = f_y = 0$  or where either  $f_x$  or  $f_y$  is undefined). In #10, such a function is called *harmonic*.
2. From Apostol, 12.4, pages 450-451, do problems 1(a,c,e), 2, 5, 12.
3. From Apostol, 12.8, pages 456-457: do problems 1(b,e), 4, 5, 13(a,b), 19.
4. From Apostol, 12.11, pages 460-462: do problems 1, 5, 12, 20. (In #20, we say that  $n$ -space together with this distance function is a *metric space*.)
5. For any complex number  $z = x + iy$  with  $x, y \in \mathbb{R}$ , the *real part* of  $z$  is  $x$  and it is denoted by  $\operatorname{Re}(z)$ . For any positive integer  $n$ , define  $f_n(x, y) = \operatorname{Re}((x + iy)^n)$ , for all real numbers  $x, y$ . Are the functions  $f_1, f_2, f_3$  harmonic? (See problem 1 above.) Any conjectures?
6. Let  $V$  be the set of continuous functions on the closed interval  $[0, 1]$ . We may add two functions by defining  $(f + g)(x)$  to be  $f(x) + g(x)$ , and we may multiply a function by a real number by defining  $(cf)(x) = c(f(x))$ .
  - a) Show that the set  $V$ , under these two operations, satisfies all the vector laws (given in and just after Theorem 12.1, page 447 of Apostol, and also listed as axioms 1-10 in Section 15.2, pages 551-552).
  - b) Show that if we define  $f \cdot g = \int_0^1 f(x)g(x)dx \in \mathbb{R}$ , then the laws of dot product (listed in Theorem 12.2 of Apostol, page 451) are also satisfied.