Math 116 Problem Set #7 Due Fri., Oct. 30, 2009, in class.

Read Apostol, Chapter 12, section 16; and Chapter 13, sections 1-16.

1. From Apostol, 13.5, page 477: do problems 2, 3, 8. From Apostol, 13.8, pages 482-483: do problems 2 and 12.

2. From Apostol, 13.11, page 487-488: do problems 1(a,e), 2(b), 3(c), 8(a).

3. Let $v_1, v_2, v_3 \in \mathbb{R}^3$. Suppose that v_1 and v_2 are non-zero orthogonal vectors, and let P be the span of $\{v_1, v_2\}$. For $i = 1, 2$ let $a_i = v_3 \cdot v_i / \|v_i\|^2$, and let $w = a_1 v_1 + a_2 v_2$.

a) Show that P is a plane through the origin.

b) Show that w is the orthogonal projection of v_3 onto P; i.e. that $v_3 - w$ is orthogonal to every vector in the plane.

c) Show that w is the closest point to v_3 on P.

d) Interpret parts (b) and (c) in the special case that v_3 lies in P, and explain why those parts were already known by a previous result in that case.

4. Suppose that $v, w \in \mathbb{R}^3$. If $v \times (v \times w) = 0$, what can you conclude about v and w? Is this a necessary and sufficient condition?

Note: In the following problems, the points of $\mathbb{R}^n$ are regarded as vectors in the usual way (with "tails at the origin"), under the usual addition and scalar multiplication in terms of coordinates.

5. Show that the points of a line in $\mathbb{R}^n$ satisfy all the laws of a vector space (see Axioms 1-10, section 15.2, pages 551-552 of Apostol) if and only if the line contains the origin.

6. a) Let L be a line in $\mathbb{R}^2$. Prove that L (as a subset of $\mathbb{R}^2$) spans $\mathbb{R}^2$ if and only if L does not contain the origin. Can the set L ever be linearly independent?

b) State and prove an analog for planes in $\mathbb{R}^3$.

c) What about lines in $\mathbb{R}^3$?