Math 116 Problem Set #8 Due Fri., Nov. 6, 2009, in class.

Read Apostol, Chapter 13, sections 18-23, and Chapter 14, sections 1-5.

1. From Apostol, 13.14, pages 491-492: do problems 1(c), 2, 3; and from 13.21, page 503: do problems 6, 7, 11, 12.

2. From Apostol, 13.24, pages 508-509: do problems 8, 21, 32; and from 13.25, pages 509-511: do problems 1, 8, 19.

3. From Apostol, 14.4, pages 516-517: do problems 1, 9, 14, 19.

4. In analogy with section 9.4 of Apostol, write a vector $(a, b, c, d) \in \mathbb{R}^4$ as $a+bi+cj+dk$. Also, define a multiplication law on $\mathbb{R}^4$ so that $ij = k$, $jk = i$, $ki = j$, $ji = -k$, $kj = -i$, $ik = -j$ (as with cross product); and also so that $i^2 = j^2 = k^2 = -1$ (unlike the cross product); and so that $1v = v = v1$ for all $v \in \mathbb{R}^4$.

a) Under this multiplication, evaluate $(i + j)^2$ and $(1 + i + j + k)(1 - i - j - k)$.

b) Which of the axioms of a field (Axioms 1-6 on page 18 of Apostol) are satisfied by the elements of $\mathbb{R}^4$ under vector addition and the above multiplication law?

c) Call an element $v \in \mathbb{R}^4$ *central* if $vw = wv$ for all $w \in \mathbb{R}^4$. Find all the central elements of $\mathbb{R}^4$.

5. a) Let $L$ be a line in the plane and let $C$ be a conic section in the plane. At how many points can $L$ and $C$ meet? Give examples illustrating each possible value.

b) In part (a), if $L$ is tangent to $C$ at a point $P$, then at how many points (including $P$) can $L$ and $C$ meet?

c) Make a conjecture concerning the number of points at which two distinct conic sections $C, C'$ in the plane can meet. Give examples to illustrate each of the possible values.

6. Suppose that $F : \mathbb{R} \to \mathbb{R}^2$ is a differentiable vector-valued function, that $c \in \mathbb{R}$, and that $\int_0^x F(t)\,dt = (x^2 + x, e^x + c)$ for all $x \in \mathbb{R}$. Find $F$ and find $c$.