Math 116 Problem Set #9 Due Fri., Nov.13, 2009, in class.

Reminder: The second exam is on Monday, November 9 in class. One two-sided handwritten 5"x 7" index card is permitted during the exam, but no other aids.

Read Apostol, Chapter 14, sections 6-20.

1. From Apostol, 14.7, pages 524-525: do problems 3, 5, 7, 17; and from 14.9, pages 528-529: do problems 3, 5, 7.

2. From Apostol, 14.13, pages 535-536: do problems 1, 11, 13; from 14.15, pages 538-539: do problems 1 (do just for #3,5 of 14.9), 2, 7; and from 14.19, pages 543-545: 1, 2(a), 4, 8.

3. Let $F : \mathbb{R} \to \mathbb{R}^n$ be a differentiable vector-valued function that parametrizes the motion of a particle in $\mathbb{R}^n$ whose speed is always at most c (where c is some positive real number).

a) Prove that if $a < b$ then $\|F(b) - F(a)\| \leq c(b - a)$. Also explain why this is reasonable from a geometric point of view.

b) Give an example of a function F and values $a < b$ for which there is equality in part (a), and give another example in which there is a strict inequality.

4. Let $f : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function. Parametrize the plane curve $y = f(x)$ by $F(t) = (t, f(t))$, and suppose that $F(a) = (a, f(a))$ is an inflection point of this curve for some value of a. Prove that $T'(a) = 0$, where $T(t)$ is the unit tangent vector to the curve at the point $F(t)$. Is the principal normal vector $N(a)$ at $F(a)$ defined?

5. a) Find the arclength of the plane curve given parametrically by $F(t) = (2t, \frac{t^3}{3} + \frac{1}{t})$, for $1 \leq t \leq 3$.

b) Find the arclength of the plane curve whose graph is $y = \log \cos x$ for $0 \leq x \leq \pi/4$. (Here log is the natural logarithm.)

6. Consider the curve in $\mathbb{R}^3$ given parametrically by $F(t) = ti + t^2 j + t^3 k$, where i, j, k are the unit basis vectors. Find the curvature at the origin, and find all points where the curvature is zero.