Math 116 Problem Set #10 Due Fri., Nov. 20, 2009, in class.

Read Apostol, Chapter 8, sections 1-13.

1. From Apostol, 8.5, pages 311-312, do problems 1-6, 10.

2. From Apostol, 8.7, pages 319-322, do problems 1, 2, 4, 8, 9, 18.

3. From Apostol, 8.14, pages 328-329, do problems 3-6, 13, 14, 18.

4. Let $a(x), b(x), c(x)$ be twice-differentiable functions $\mathbb{R} \to \mathbb{R}$.

a) Show that the set of solutions to the differential equation $y'' + a(x)y' + b(x)y = c(x)$ satisfies the axioms of a vector space (see §15.2) if and only if $c(x)$ is the constant function 0.

b) Show that if $a(x) = 0$, $b(x) = 1$, and $c(x) = 0$, then $\sin(x)$ and $\cos(x)$ are linearly independent elements of this vector space.

5. Solve the initial value problem given by $y''' - y'' - 2y' = e^x$, $y(0) = 0$, $y'(0) = 1$, $y''(0) = 3$.

6. a) Say a monetary quantity (e.g. an interest-bearing deposit, or the consumer price index) grows at a fixed rate of r percent per year, compounded continuously. Let C be the value at time $t = 0$. Write an initial value problem that corresponds to this situation, and solve this problem, obtaining a formula for this function of t in terms of r. Find the time t_0 that it takes for the quantity to double. What is the relationship between t_0 and r? Give this both in precise form and numerically (with the value of any constant given to within .01).

b) In the situation of (a), suppose that at the end of each year, the quantity is i percent higher than at the beginning of that year. Find the relationship between i and r. Also give a precise expression for t_0 in terms of i. If $t_0 = k/i$, what is the numerical value of k (to within .01) if $i = 4$? 8? 16? How does k vary with i? What is the limit of k as $i \to 0$? as $i \to \infty$?

[Note: in part (a), r is the "interest rate before compounding"; in part (b), i is the "effective annual yield."]