Math 116 Problem Set #11 Due Mon., Nov. 30, 2009, in class.

Read Apostol, Chapter 8, sections 15-27.

1. From Apostol, 8.17, pages 333-334, do problems 3, 4, 8, 9.

2. From Apostol, 8.19, page 339, do problems 1, 3, 7.

3. From Apostol, 8.22, page 344, do problems 4 and 5; and 8.24, page 347, problems 1, 14.

4. From Apostol, 8.26, page 350, do problem 4; and 8.28, page 355, problem 2.

5. a) From Apostol, 13.25, page 510, do problem 20.

b) In this situation, determine which of these curves are ellipses and which are hyperbolas. Find the family of orthogonal trajectories to the family of ellipses with these foci.

6. For any polynomial $L = \sum_{i=0}^{n} a_i D^i$ in a variable D, and any n-times differentiable function f, define $Lf = \sum_{i=0}^{n} a_i f^{(i)}$. (Here $f^{(0)}$ means f.)

a) Show that if $L = L_1 L_2$ (as polynomials in D), then $Lf = L_1(L_2 f)$.

b) Show that if $L = L_1 L_2$ then the solutions to the linear homogeneous differential equation $L_2 f = 0$ are also solutions to the linear homogeneous differential equation $Lf = 0$. Show moreover that the solutions to $L_2 f = 0$ form a vector subspace of the solutions to $Lf = 0$ (i.e. a subset that is also a vector space). Do the same for the linear homogeneous differential equation $L_1 f = 0$ instead of $L_2 f = 0$. [Hint: $L_1 L_2 = L_2 L_1$.]

c) Using induction, show that for any constant a, the functions $e^{ax}, xe^{ax}, \ldots, x^{n-1}e^{ax}$ are solutions to the linear homogeneous differential equation $(D-a)^n f = 0$.

d) Consider a polynomial $L = \prod_{i=1}^{m} (D - a_i)^{r_i}$ of degree n, where $a_1, \ldots, a_m$ are distinct constants, where each $r_i \geq 1$, and where $\sum_{i=1}^{m} r_i = n$. Show that the n functions $x^j e^{a_i x}$, for $1 \leq i \leq m$ and $0 \leq j \leq r_i - 1$, lie in the vector space of solutions to the linear homogeneous differential equation $Lf = 0$.

e) Find the general solution to the linear homogeneous differential equation

$$f^{(4)} - 2f'' + f = 0.$$