Math 116 Problem Set #12 Due Fri., Dec. 4, 2009, in class.

Read Apostol, Chapter 15.

1. From Apostol, 15.5, page 555, do problems 1, 4, 5, 13, 14; and from 15.9, page 560, do problems 5-7, 9, 14.

2. From Apostol, 15.12, pages 566-568, do problems 1(b,e), 4, 8, 13(a,b,d); and from 15.16, page 576, do problems 1(a), 2(b), 4.

3. Prove that the functions e^x, e^{2x}, e^{3x} are linearly independent in the real vector space V consisting of differentiable functions.

4. Let V be the set of solutions to the differential equation $f'(x) = f(x)$ and let W be the set of solutions to the differential equation $f''(x) - 3f'(x) + 2f(x) = 0$.

a) Show that V and W are real vector spaces, and that V is a subspace of W.

b) Find a basis for V, and the dimension of V.

c) Extend your basis of V to a basis of W (i.e. find a basis of W that contains your basis of V), and find the dimension of W.

5. Let $W \subset \mathbb{R}^3$ be the subspace given by $x + y + z = 0$. Find a basis of W and extend it to a basis of $\mathbb{R}^3$.

6. Prove or disprove each of the following assertions:

a) If V is a finite dimensional vector space with basis $B = \{v_1, \ldots, v_n\}$, and W is a subspace of V, then $B \cap W$ is a basis for W.

b) If V is a vector space, and S is a linearly independent subset of V that is not contained in *any* strictly larger linearly independent subset of V, then S is a basis of V.