This optional problem set may be handed in for credit until noon on Monday, Dec. 14. Your best 11 problem sets will count toward your grade.

Read Apostol, Chapter 16.

1. From Apostol, 16.4, pages 582-583, do problems 1, 4, 7 (where V_2 denotes $\mathbb{R}^2$), 25 (see instructions before problem 24); and from 16.8, pages 589-590, do problems 3, 6, 9 (again V_2 denotes $\mathbb{R}^2$), 23.

2. From Apostol, 16.12, pages 596-597, do problems 1, 2(b), 3(a,b) (again V_2 denotes $\mathbb{R}^2$), 16 (see instructions before problem 11); from 16.16, pages 603-604, do problems 6, 7; and from 16.20, pages 613-614, do problems 1, 11, 12.

3. Let $S : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that takes e_1 to $e_1 + e_2$ and takes e_2 to $e_1 + 2e_2$. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that takes e_1 to $e_1 - e_2$ and takes e_2 to e_1. Let $U = S \circ T$.

a) Evaluate $U(e_1)$ and $U(e_2)$.

b) Find the matrices A, B, C associated to the linear transformations S, T, U.

c) Verify that $C = AB$.

4. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation given by $T(x, y) = (u, v)$, where $(u, v) = (x + y, x - y)$.

a) Find the inverse linear transformation S of T, giving $S(u, v)$ explicitly. [Hint: Express x, y in terms of u, v.]

b) Find the matrices A, B of S, T and explicitly verify that they are inverses.

5. For each of the following linear transformations $\mathbb{R}^2 \to \mathbb{R}^2$, find where e_1 and e_2 are sent, and find the matrix associated to the transformation.

a) $v \mapsto -v$.

b) v is sent to its reflection through the x axis.

c) v is rotated counterclockwise through an angle of θ. (Here θ is a fixed angle between 0 and 2π.)

6. Let V be the vector space of infinitely differentiable functions on the real line, and define $T : V \to V$ by $T(f) = f'' - 3f' + 2f$.

a) Verify that T is a linear transformation, and find a basis for its nullspace $N(T)$.

b) Find an isomorphism $N(T) \to \mathbb{R}^2$.

7. Let M_2 be the vector space of 2×2 matrices. Let A be the matrix corresponding to the linear transformation T that takes e_1 to e_2 and takes e_2 to $e_1 + e_2$. For any matrix B in M_2, define $S(B) = AB$.

a) Show that the map $S : M_2 \to M_2$ is a linear transformation.

b) Pick a basis for M_2 and write the matrix for S under this basis. (Caution: S is a linear transformation on M_2, *not* on $\mathbb{R}^2$.)