Math 603 Sample Exam Spring 2005

For each of the following, either give an example or explain why none exists.

1. An integral domain R and a non-0 R-module M such that $M^* := \mathrm{Hom}(M, R)$ is 0.

2. A ring R and an R-module M that is torsion-free but not free.

3. A ring R, an R-module M, and a surjective homomorphism $N' \to N$ of R-modules such that $M \otimes_R N' \to M \otimes_R N$ is not surjective.

4. A positive integer n such that $\mathbb{Z}[1/n]$ is not flat over $\mathbb{Z}$.

5. Two $\mathbb{Z}$-modules M, N such that $\mathrm{Tor}^1(M, N) \neq 0$.

6. An integral domain that is integrally closed but is not integral over $\mathbb{Z}$.

7. A finitely presented flat module M over the p-adic integers $\mathbb{Z}_p$ such that M is not free.

8. A polynomial $f(x, y) \in \mathbb{Z}[x, y]$ and an ideal $I \subset R := \mathbb{Z}[x, y]/(f(x, y))$ such that I is not finitely generated as an R-module.

9. A Noetherian ring that is not an Artin ring.

10. A discrete valuation ring R of characteristic 3.

11. A Noetherian ring R and a principal prime ideal $I \subset R$ of height 2 in R.

12. A field of order 50.

13. A field K and a polynomial $f(x) \in K[x]$ of degree > 0 such that $f'(x) = 0 \in K[x]$.

14. A finite field extension of $\mathbb{Q}$ that is not Galois over $\mathbb{Q}$.

15. A field extension of $\mathbb{R}$ of degree 7.

16. A Galois extension of $\mathbb{C}(x)$ of degree 10.

17. A normal basis of $\mathbb{Q}(\sqrt{3})$ over $\mathbb{Q}$.

18. A cyclic field extension $K \subset L$ and an element $\alpha \in L$ such that $\alpha \notin K$ and such that the norm of α is 1.

19. A finite field extension K of $\mathbb{Q}$ such that there are infinitely many field homomorphisms $K \to \mathbb{C}$.

20. A positive integer n such that the regular n-gon cannot be constructed by straightedge and compass.