

## LIST OF PUBLICATIONS

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### Research articles

**16. MULTITERMINAL NETWORK TOMOGRAPHY** (with F. Alberto Grünbaum), in preparation.

*Abstract:* We identify a large class of directed graphs (or multiterminal networks) for which a very general inverse problem can be solved explicitly. These generalizes previous results of the authors concerning simple models for diffuse or optical tomography.

**15. HOLONOMIC HORN  $D$ -MODULES** (with Alicia Dickenstein), in preparation.

*Abstract:* We study Horn systems in their binomial formulation. We characterize which of these systems are holonomic for generic parameters. In this case we compute the generic holonomic rank, and write down an explicit basis of solutions in terms of associated  $A$ -hypergeometric functions.

**14. HYPERDISCRIMINANTS** (with Amit Khetan), in preparation.

*Abstract:* Given  $d$  and  $n$  we consider all the monomials of degree  $d$  on the solutions  $x_1, \dots, x_n$  of an algebraic equation of degree  $n$ . The hyperdiscriminant is the polynomial on the coefficients of this algebraic equation that vanishes exactly when two (or more) of the aforementioned monomials vanish. We study the irreducible factors of the hyperdiscriminant, and relate their Newton polytopes to the Newton polytope of the ordinary discriminant.

**13. AN IDENTIFICATION PROBLEM FOR MULTITERMINAL NETWORKS: SOLVING FOR THE TRAFFIC MATRIX FROM INPUT-OUTPUT MEASUREMENTS**, (with F. Alberto Grünbaum), submitted.

*Abstract:* We construct a Markov chain that models the traffic of messages in a multiterminal network consisting of *input*, *intermediate* and *output* terminals. The topology of the network is assumed to be known, but the Markovian routing strategy is not known. We solve the problem of determining the unknown one-step transition probability matrix of our random walk from input-output measurements of “travel time”. We give explicit inversion formulas (up to a natural gauge) in a nontrivial example. The result holds for a large (but not arbitrary) class of multiterminal networks, many of which are indicated here. The networks that we display here are constructed in a canonical fashion from certain graphs. Some of these graphs as well as the way to go from graphs to networks are also displayed here. One example of a graph for which our method works is the edge graph of a hypercube in any dimension.

**12. HOMOLOGICAL METHODS FOR HYPERGEOMETRIC FAMILIES**, (with Ezra Miller and Uli Walther), [arXiv:mathAG/0406386](https://arxiv.org/abs/mathAG/0406386), submitted.

*Abstract:* We analyze the behavior of the holonomic rank in families of holonomic systems over complex algebraic varieties by providing homological criteria for rank-jumps in this general setting. Then we investigate rank-jump behavior for hypergeometric systems  $H_A(\beta)$  arising

from a  $d \times n$  integer matrix  $A$  and a parameter  $\beta \in \mathbb{C}^d$ . To do so we introduce an Euler–Koszul functor for hypergeometric families over  $\mathbb{C}^d$ , whose homology generalizes the notion of a hypergeometric system, and we prove a homology isomorphism with our general homological construction above. We show that a parameter  $\beta \in \mathbb{C}^d$  is rank-jumping for  $H_A(\beta)$  if and only if  $\beta$  lies in the Zariski closure of the set of  $\mathbb{Z}^d$ -graded degrees  $\alpha$  where the local cohomology  $\bigoplus_{i < d} H_m^i(\mathbb{C}[\mathbb{N}A])_\alpha$  of the semigroup ring  $\mathbb{C}[\mathbb{N}A]$  supported at its maximal graded ideal  $\mathfrak{m}$  is nonzero. Consequently,  $H_A(\beta)$  has no rank-jumps over  $\mathbb{C}^d$  if and only if  $\mathbb{C}[\mathbb{N}A]$  is Cohen–Macaulay of dimension  $d$ .

**11. ARBITRARY RANK JUMPS FOR  $A$ -HYPERGEOMETRIC SYSTEMS THROUGH LAURENT POLYNOMIALS**, (with Uli Walther), [arXiv:mathCO/0404183](https://arxiv.org/abs/mathCO/0404183), submitted.

*Abstract:* We investigate the solution space of hypergeometric systems of differential equations in the sense of Gelfand, Graev, Kapranov and Zelevinsky. For any integer  $d \geq 2$  we construct a matrix  $A_d \in \mathbb{N}^{d \times 2d}$  and a parameter vector  $\beta_d$  such that the holonomic rank of the  $A$ -hypergeometric system  $H_{A_d}(\beta_d)$  exceeds the simplicial volume  $\text{vol}(A_d)$  by at least  $d-1$ . The largest previously known gap between rank and volume was two. Our argument is elementary in that it uses only linear algebra, and our construction gives evidence to the general observation that rank-jumps seem to go hand in hand with the existence of multiple Laurent (or Puiseux) polynomial solutions.

**10. COMBINATORICS OF RANK JUMPS IN SIMPLICIAL  $A$ -HYPERGEOMETRIC SYSTEMS**, (with Ezra Miller), [arXiv:mathAC/0402071](https://arxiv.org/abs/mathAC/0402071), submitted.

*Abstract:* Let  $A$  be an integer  $d \times n$  matrix and assume that the convex hull  $\text{conv}(A)$  of its columns is a simplex of dimension  $d-1$ . It is known that the semigroup ring  $\mathbb{C}[\mathbb{N}A]$  is Cohen–Macaulay if and only if the rank of the GKZ hypergeometric system  $H_A(\beta)$  equals the normalized volume of  $\text{conv}(A)$  for all complex parameters  $\beta \in \mathbb{C}^d$ . Our refinement here shows that  $H_A(\beta)$  has rank strictly larger than the volume of  $\text{conv}(A)$  if and only if  $\beta$  lies in the Zariski closure (in  $\mathbb{C}^d$ ) of all  $\mathbb{Z}^d$ -graded degrees where the local cohomology  $\bigoplus_{i < d} H_m^i(\mathbb{C}[\mathbb{N}A])$  is nonzero.

**9. BIVARIATE HYPERGEOMETRIC  $D$ -MODULES** (with Alicia Dickenstein and Timur Sadykov), to appear in *Advances in Mathematics*.

*Abstract:* We undertake the study of bivariate Horn systems for generic parameters. We prove that these hypergeometric systems are holonomic, and we provide an explicit formula for their holonomic rank as well as bases for their spaces of complex holomorphic solutions. We also obtain analogous results for the generalized hypergeometric systems arising from lattices of any rank.

**8. A NETWORK TOMOGRAPHY PROBLEM RELATED TO THE HYPERCUBE** (with F. Alberto Grünbaum), *Contemporary Mathematics*, **362** (2004), pp. 189–197.

*Abstract:* We construct a Markov chain whose state space is built out of the hypercube. We solve the problem of determining the unknown one-step transition probability matrix of our random walk from input-output measurements of “time of flight”. This construction is inspired by a discrete model of diffuse tomography, but we formulate this as a general problem for a hypercube of arbitrary dimension. We give explicit inversion formulas (up to a natural gauge) in the case of dimension four, but the result holds for any dimension.

**7. TRANSFORM METHODS FOR THE HYPERGEOMETRIC DISTRIBUTION** (with Ian Dinwoodie and Ed Mosteig), *Statistics and Computing*, **14** (2004) pp. 287–297.

*Abstract:* Two new methods for computing with hypergeometric distributions on lattice points are presented. One uses Fourier analysis, and the other uses Gröbner bases in the Weyl algebra. Both are very general and apply to log-linear models that are graphical or non-graphical.

**6. EXCEPTIONAL PARAMETERS FOR GENERIC  $A$ -HYPERGEOMETRIC SYSTEMS**, *International Mathematics Research Notices* (2003) pp. 1225–1248.

*Abstract:* The holonomic rank of an  $A$ -hypergeometric system  $H_A(\beta)$  is conjectured to be independent of the parameter vector  $\beta$  if and only if the toric ideal  $I_A$  is Cohen Macaulay. We prove this conjecture in the case that  $I_A$  is generic by explicitly constructing more than  $\text{vol}(A)$  many linearly independent hypergeometric functions for parameters  $\beta$  coming from embedded primes of certain initial ideals of  $I_A$ . We also show that, under no assumptions on the toric ideal  $I_A$ , the exceptional set is Zariski constructible, and propose an algorithm to compute it.

**5. A NONLINEAR INVERSE PROBLEM INSPIRED BY THREE-DIMENSIONAL DIFFUSE TOMOGRAPHY: EXPLICIT FORMULAS** (with F. Alberto Grünbaum), *International Journal of Imaging Systems & Technology* **12** (2002) pp. 198–203.

*Abstract:* We use time of flight information to obtain explicit inversion formulas for a simple model of optical tomography. This can be considered as an instance of a more general nonlinear inversion problem for multiterminal networks. In this case the network is nonplanar with lots of cycles but the underlying physical model yields a highly structured situation.

**4. EXPLICIT INVERSION FORMULAS FOR A MODEL IN DIFFUSE TOMOGRAPHY** (with F. Alberto Grünbaum), *Advances in Applied Mathematics* **29** (2002) 172–183.

*Abstract:* We study a polynomial system arising from a two dimensional model in diffuse tomography. The most remarkable feature of this system is that, although it is highly nonlinear, it can be solved by a sequence of linear steps. This allows us to explicitly solve the inverse problem for our model, up to a natural “gauge transformation”.

**3. RANK JUMPS IN CODIMENSION 2  $A$ -HYPERGEOMETRIC SYSTEMS**, *Journal of Symbolic Computation*, Special Issue on Effective Methods in Rings of Differential Operators, **32** (2001), pp. 619–641.

*Abstract:* The holonomic rank of an  $A$ -hypergeometric system  $H_A(\beta)$  is shown to depend on the parameter vector  $\beta$  when the underlying toric ideal is a non Cohen Macaulay codimension 2 toric ideal. The set of exceptional parameters is usually infinite.

**2. RATIONAL SUMMATION OF RATIONAL FUNCTIONS**, *Beiträge zur Algebra und Geometrie* Vol. **41** (2000) pp. 531–536.

*Abstract:* In this article, we characterize rational functions for which their indefinite sum is again a rational function.

**1. THE DISCRETE VERSION OF THE BISPECTRAL PROBLEM** (with Fernando Levstein), *CRM Proceedings and Lecture Notes* Vol. **14** (1998) pp. 93–104.

*Abstract:* We consider a discrete version of the Bispectral Problem. In this case the solution can be given in terms of the  $q$ -Racah polynomials of Askey and Wilson. We see that an analog of the Darboux transformation preserves bispectrality. We show that applying a “dressing”

Darboux transformation has the same effect as adding a new mass at zero and we conjecture the appearance of analogs of Krall polynomials.

### Theses

- COMBINATORIAL ASPECTS OF HYPERGEOMETRIC FUNCTIONS, Ph.D. thesis under the supervision of Bernd Sturmfels, UC Berkeley, 2002.

*Abstract:* We construct exceptional parameters of  $A$ -hypergeometric systems constructed from certain toric ideals which do not satisfy Serre's  $S_2$  condition. This generalizes the results in the author's previous articles on this topic. We also study the relationship between Horn and  $A$ -hypergeometric systems, providing a link with the more classical approach to special functions. Finally, we propose an algorithm for computing the solutions of an  $A$ -hypergeometric system  $H_A(\beta)$  whose support is finite.

- POLINOMIOS ORTOGONALES Y TRANSFORMACIÓN DE DARBOUX (*Orthogonal polynomials and the Darboux transformation*), Undergraduate thesis under the supervision of Fernando Levstein, Universidad Nacional de Córdoba, Argentina, 1996.

*Abstract:* We study the problem of characterizing which families of orthogonal polynomials satisfy difference equations. This can be seen as a discrete version of the Bispectral Problem. We attempt to generalize a result of Leonard, which states that the only orthogonal polynomials that satisfy a second order difference equation are the Askey-Wilson polynomials. In this context, we investigate the effect of the Darboux transformation, and conjecture the existence of analogs of the Krall polynomials.

### Other publications

- TEORÍA DE GRAFOS, ALGORITMOS, COMPLEJIDAD COMPUTACIONAL Y OPTIMIZACIÓN COMBINATORIA (*Graph theory, algorithms, computational complexity and combinatorial optimization*) (with Oscar Porto), *Lecture Notes, B Series*, No 37/96, FaMAF, Universidad Nacional de Córdoba.

*Abstract:* These are the lecture notes from a short course given by Professor Oscar Porto, which took place in FaMAF in February 1996. The notes contain the basics on graph theory as it relates to optimization and complexity, including NP theory, Dynamic programming and Network flow problems.