MATH 1070 COURSE GOALS

1. Meta-mathematical goals:

Any math you know, you should know well enough to

- Explain it to someone else
- Use it to solve an interesting problem
- Recognize when it occurs in an application
- Write a coherent solution that someone else could learn from
- Remember it for many years, or at least significantly beyond the final exam.

Problem-solving heuristics:

check whether you understand what is being asked;
look at some data;
try to solve a special case;
try to solve a simpler problem;

Verbal and argumentation skills:

use of proper math grammar and precise terms;
recognizing what constitutes a counterexample to a hypothesis;
recognizing what constitutes a counterexample to an argument;

Free and bound variables:

Understand the difference and recognize them in contexts such as:

- in integrals ("dummy variables");
- in sums;
- in max-min problems;
- in the definition of a function

2. Big concepts:

Limits and continuity:

intuitive understanding;
ability to use the definition in tricky cases;
as a basis for other definitions such as

- Derivatives
- Improper integrals
- Continuous compunding and exponential functions
- Convergence of series
- Asymptotic statements
- Taylor’s Theorem with remainder

Functions:

how to graph them;
estimating when you have only the graph;
inverse functions: concepts, notation, units
derivative as an operator;
indefinite integral is a function of the upper limit;

Exponential behavior:

exponential growth, decay and approach;
relating infinitesimal behavior to long-term behavior;
knowing which real life models behave in these ways;

Estimating:

linear estimates with first derivative;
higher order estimates with Taylor series;
integrating something near gives something near;
computational tricks and knowing approximate constants;

Bounding:

what it means to bound a quantity in an interval;
bounding partial sums of series;
bounding series by integrals and vice versa;
convexity can make a linear estimate into a bound;

Units:

- Understand that in studying calculus, many quantities will come with units, understand how numbers change when units change, and how to manipulate expressions with units;
- Unitless expressions and time constants
- Understand a few more sophisticated features of units, such as:
    - exponential and logarithm takes unitless quantities;
    - units of df/dx are units of f divided by units of x;
    - units of \int f dx are units of f times units of x;

Orders of growth:

powers, exponentials, logs and their relations at 0 and infinity;
Notations o(.), O(.) and ~ ;
nth order approximation at a point;

3. Further detailed content

Sequences and series:

Writing them when given them informally;

Limits of sequences;
Convergence of series means convergence of partial sums;
Summing geometric series and comparing other sums to these;

Derivatives:

Definitions, notations and interpretations

- Concept of instantaneous rate
- Formal definition via limits
- Pictorial definition via slope
- Interpretation as marginal effect

Relation of graph to first and second derivatives
Relation to analytic geometry: tangent lines and secant lines
Computing: powers, basic transcendental functions, product/quotient rule, chain rule, inverse funtions

Revisiting limits with derivatiaves:

indeterminate forms and L'Hôpital's rule
formalizing asymptotic statments

Integration:

Riemann sums: writing them, comparing to integrals;
Indefinite integral as a function of the upper limit;
Computation of integrals: remember/guess, substitution, integration by parts
Interpretations and applications:

- area
- signed area
- volume
- probability
- total amounts
- averages

Improper integrals: meaning, formal definition, interpretation;

Taylor polynomials:

Taylor polynomials as best nth order approximation
Taylor's theorem

- exact remainder term at undetermined intermediate point
- MVT as a special case
- remainder bounds

Computing Taylor polynomials

- formula for nth term
- sums and products of series
- composition of series

Optimization:

terminology: minimum, maximum, extremum (and their plurals), critical point, global versus local;

why extrema occur at critical points

role of endpoints

role of continuity and differentiability

what you can learn from second derivatives