

Homework II: Math 546 version

Read through sections A1, A2, A4, 1.1 and 1.2. These are sections we will not cover in class any better than we already have, but that give some details you'll need to know to understand completely the formal infrastructure.

Now do the following problems. Since the point is often to prove measurability, do not assume it! But you may use results from anywhere in the text.

1. Durrett Chapter 1 Exercise 2.4
2. Durrett Appendix Exercise 4.3
3. Read the statement of the π - λ theorem on page 444 and then do Exercises 2.1 of the Appendix and 4.4 of Chapter 1.
4. Durrett Chapter 1 Exercise 5.3.
5. Extra credit: This could be considered part (iii) of Durrett, Chapter 1, Exercise 5.3. Suppose further that f is bounded, specifically $|f| \leq L$. Assume for simplicity that $\int_0^1 f = 0$. Use the methods of Problem 7 on the first homework set to get an estimate $\mathbb{P}(|I_n| \geq a/\sqrt{n})$. Which is better, the bound in (ii) or this new bound, and why would you expect this? [Hints: (1) show that $\mathbb{E}e^{\lambda X_1}$ is maximized when the distribution is $(1/2)\delta_L + (1/2)\delta_{-L}$; (2) try using $\lambda = a/\sqrt{n}$ and use inequalities to eliminate the hyperbolic cosine in the upper bound.]