Stat 531 / Math 547, Spring 2005, Homework Set II

Due Friday February 16, 2007

1. Durrett Exercise 2.11. (Convergence criterion)

2. Durrett Exercise 3.2. (Infinitely many back and forths)

3. Durrett Chapter 4, Exercise 3.9, 3.10 and 3.11 - I have listed these together to give you more flexibility as to how to organize your answser.

4. Suppose $\{X_n\}$ is a martingale with increments $X_{n+1} - X_n$ bounded in absolute value by M. Can X_n converge with positive probability on the event $\{A_\infty = \infty\}$, where A is the action process from page 255? Can you answer this question without the assumption of bounded increments?

5. Let $\{X_n\}$ be a martingale with $\mathbf{E}X_n^2 < \infty$ for all n. Let M_n an A_n be the martingale and action processes from the Doob decomposition of X_n^2, as in the last part of Section 4.4. Define

$$Q_n = \sum_{k=1}^{n} (X_k - X_{k-1})^2 \,,$$

the so-called quadratic variation process. Prove or find a counterexample:

   (a) the event $\{A_\infty = \infty\}$ implies the event $\{Q_n \to \infty\}$ up to a set of measure zero (in other words, $\mathbf{P}(A_\infty = \infty, Q_\infty < \infty) = 0$);

   (b) the event $\{A_\infty < \infty\}$ implies the event $\{Q_n\}$ is bounded, up to a set of measure zero (in other words, $\mathbf{P}(A_\infty < \infty, Q_\infty = \infty) = 0$);

   (c) the event $\{A_\infty = \infty\}$ implies the event $\{Q_n \to \infty\}$ up to a set of measure zero, assuming that the increments $|X_n - X_{n-1}|$ are bounded by some constant M.