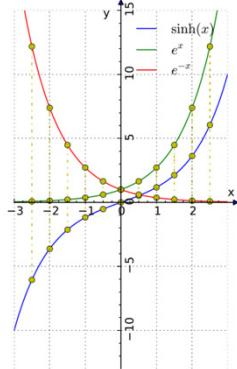
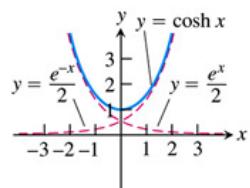


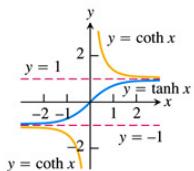
**Hyperbolic sine:**

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

**Hyperbolic cosine:**

$$\cosh x = \frac{e^x + e^{-x}}{2}$$



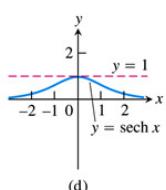


(c)  
**Hyperbolic tangent:**

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

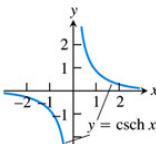
**Hyperbolic cotangent:**

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$



(d)  
**Hyperbolic secant:**

$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}} \quad \operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$$



(e)  
**Hyperbolic cosecant:**

**TABLE 7.5** Derivatives of hyperbolic functions

$$\begin{aligned}\frac{d}{dx}(\sinh u) &= \cosh u \frac{du}{dx} \\ \frac{d}{dx}(\cosh u) &= \sinh u \frac{du}{dx} \\ \frac{d}{dx}(\tanh u) &= \operatorname{sech}^2 u \frac{du}{dx} \\ \frac{d}{dx}(\coth u) &= -\operatorname{csch}^2 u \frac{du}{dx} \\ \frac{d}{dx}(\operatorname{sech} u) &= -\operatorname{sech} u \tanh u \frac{du}{dx} \\ \frac{d}{dx}(\operatorname{csch} u) &= -\operatorname{csch} u \coth u \frac{du}{dx}\end{aligned}$$

**TABLE 7.6** Integral formulas for hyperbolic functions

$$\begin{aligned}\int \sinh u \, du &= \cosh u + C \\ \int \cosh u \, du &= \sinh u + C \\ \int \operatorname{sech}^2 u \, du &= \tanh u + C \\ \int \operatorname{csch}^2 u \, du &= -\coth u + C \\ \int \operatorname{sech} u \tanh u \, du &= -\operatorname{sech} u + C \\ \int \operatorname{csch} u \coth u \, du &= -\operatorname{csch} u + C\end{aligned}$$

**TABLE 7.4** Identities for hyperbolic functions

$$\begin{aligned}\cosh^2 x - \sinh^2 x &= 1 \\ \sinh 2x &= 2 \sinh x \cosh x \\ \cosh 2x &= \cosh^2 x + \sinh^2 x \\ \cosh^2 x &= \frac{\cosh 2x + 1}{2} \\ \sinh^2 x &= \frac{\cosh 2x - 1}{2} \\ \tanh^2 x &= 1 - \operatorname{sech}^2 x \\ \coth^2 x &= 1 + \operatorname{csch}^2 x\end{aligned}$$

Why "hyperbolic"?

Math 103 – Rimmer  
7.3 Hyperbolic Trig.  
Functions

Where is this used?

**Hanging cables** Imagine a cable, like a telephone line or TV cable, strung from one support to another and hanging freely. The cable's weight per unit length is a constant  $w$  and the horizontal tension at its lowest point is a *vector* of length  $H$ . If we choose a coordinate system for the plane of the cable in which the  $x$ -axis is horizontal, the force of gravity is straight down, the positive  $y$ -axis points straight up, and the lowest point of the cable lies at the point  $y = H/w$  on the  $y$ -axis (see accompanying figure), then it can be shown that the cable lies along the graph of the hyperbolic cosine

$$y = \frac{H}{w} \cosh \frac{w}{H} x.$$

Math 103 – Rimmer  
7.3 Hyperbolic Trig.  
Functions

**Laplace's Equation**

$$u_{xx} + u_{yy} = 0$$

Used in many areas of physics:

- electromagnetic theory
- heat transfer
- fluid dynamics
- special relativity

Such a curve is sometimes called a **chain curve** or a **catenary**, the latter deriving from the Latin *catena*, meaning "chain."