12.3 The Integral Test

If $f(x)$ is: $a)$ on the interval $[k,\infty)$ constant $k>0$

$b)$

$c)$ and

,then the series $\sum_{n=k}^{\infty} a_n$ $\left(\text{with } a_n = f(n)\right)$

$i)$ is _________ when $\int_k^{\infty} f(x)\,dx$ is __________.

$ii)$ is _________ when $\int_k^{\infty} f(x)\,dx$ is _________.

Note:
the function does not necessarily have to be decreasing for all $x \in [k,\infty)$
as long as the function is decreasing "eventually"
(there is some number N so that f is decreasing for all $x > N$)

The next two slides give you a feeling of **how** the integral test works.

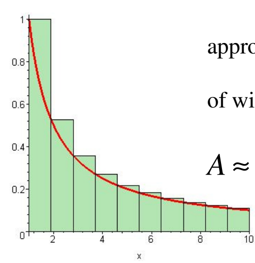
$f(x) = \dfrac{1}{x}$

on $[1,\infty)$

$a)$ continuous,

$b)$ positive,

$c)$ and decreasing



approximate the area $\int_1^{\infty} \frac{1}{x}\,dx$ with rectangles of width 1 using the left endpoint

$$A \approx 1(1) + 1\left(\tfrac{1}{2}\right) + 1\left(\tfrac{1}{3}\right) + \cdots = 1 + \tfrac{1}{2} + \tfrac{1}{3} + \cdots$$

$A \approx \sum_{n=1}^{\infty} \frac{1}{n}$ but this is an _________

$$\Rightarrow \int_1^{\infty} \frac{1}{x}\,dx \;\square\; \sum_{n=1}^{\infty} \frac{1}{n}$$

But $\int_1^{\infty} \frac{1}{x}\,dx = \lim_{b\to\infty} \int_1^{b} \frac{1}{x}\,dx$

The integral $\int_1^{\infty} \frac{1}{x}\,dx$ ________ and $\int_1^{\infty} \frac{1}{x}\,dx < \sum_{n=1}^{\infty} \frac{1}{n}$

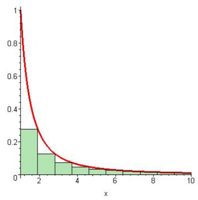
$\Rightarrow$ The series $\sum_{n=1}^{\infty} \frac{1}{n}$ must also __________.

the _______________

$f(x)=\dfrac{1}{x^2}$

on $[1,\infty)$

$a)$ continuous,
$b)$ positive,
$c)$ and decreasing



approximate the area $\int_1^\infty \frac{1}{x^2}\,dx$ with rectangles of width 1 using the right endpoint

$$A\approx 1\left(\tfrac{1}{4}\right)+1\left(\tfrac{1}{9}\right)+1\left(\tfrac{1}{16}\right)+\cdots=\tfrac{1}{4}+\tfrac{1}{9}+\tfrac{1}{16}+\cdots$$

$$A\approx \sum_{n=2}^{\infty}\frac{1}{n^2}$$ but this is an ______________

$$\Rightarrow \sum_{n=2}^{\infty}\frac{1}{n^2}<\int_1^\infty \frac{1}{x^2}\,dx \quad \Rightarrow 1+\sum_{n=2}^{\infty}\frac{1}{n^2}<1+\int_1^\infty \frac{1}{x^2}\,dx$$

$$\Rightarrow \sum_{n=1}^{\infty}\frac{1}{n^2}<1+\int_1^\infty \frac{1}{x^2}\,dx$$

But $\int_1^\infty \frac{1}{x^2}\,dx=\lim_{b\to\infty}\int_1^b x^{-2}\,dx$

The integral $\int_1^\infty \frac{1}{x^2}\,dx$ _________ and $\Rightarrow \sum_{n=1}^{\infty}\frac{1}{n^2}<1+1=2$

(The sequence of partial sums S_n is a bounded increasing sequence $\Rightarrow$ this sequence converges)

$\Rightarrow$ The series $\sum_{n=1}^{\infty}\frac{1}{n^2}$ also _________.

$f(x)=\dfrac{1}{x^p}$

on $[1,\infty)$

$a)$ continuous,
$b)$ positive,
$c)$ and decreasing

For what values of p does the integral converge?

$$\int_1^\infty \frac{1}{x^p}\,dx=\lim_{b\to\infty}\int_1^b x^{-p}\,dx$$

need $-p+1$ to be ________ so that we can get convergence by moving the $x-$term to the ____________

corresponding to this function is the series $\sum_{n=1}^{\infty}\frac{1}{n^p}$

this is called a __________.

$i)$ $\sum_{n=1}^{\infty}\frac{1}{n^p}$ **converges** when ______

$ii)$ $\sum_{n=1}^{\infty}\frac{1}{n^p}$ **diverges** when _______

Which of these converge?

$a)\ \sum_{n=1}^{\infty} \frac{1}{n^{5/2}} \qquad b)\ \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \qquad c)\ \sum_{n=1}^{\infty} \frac{3}{2n^3} \qquad d)\ \sum_{n=1}^{\infty} n^{-e}$

$a)\ \sum_{n=1}^{\infty} \frac{1}{n^{5/2}}$

$b)\ \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

$c)\ \sum_{n=1}^{\infty} \frac{3}{2n^3}$

$d)\ \sum_{n=1}^{\infty} n^{-e}$

Which of these converge?

$a)\ \sum_{n=1}^{\infty} \frac{1}{n^2+4} \qquad b)\ \sum_{n=2}^{\infty} \frac{1}{n \ln n} \qquad c)\ \sum_{n=2}^{\infty} \frac{\ln n}{n^2}$

$a)\ \sum_{n=1}^{\infty} \frac{1}{n^2+4}$

so, $\sum_{n=1}^{\infty} \frac{1}{n^2+4}$ ________ by the ___________.

Which of these converge?

$$a)\ \sum_{n=1}^{\infty}\frac{1}{n^2+4} \qquad b)\ \sum_{n=2}^{\infty}\frac{1}{n\ln n} \qquad c)\ \sum_{n=2}^{\infty}\frac{\ln n}{n^2}$$

$$b)\ \sum_{n=2}^{\infty}\frac{1}{n\ln n}$$

so, $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n\ln n}$ __________ by the ____________.

Which of these converge?

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$$c)\ \sum_{n=2}^{\infty}\frac{\ln n}{n^2}$$

so, $\displaystyle\sum_{n=2}^{\infty}\frac{\ln n}{n^2}$ __________ by the __________.