12.5 Alternating Series Test

An _______________ is of the form $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (-1)^{n-1} b_n$ or $\sum_{n=1}^{\infty} (-1)^n b_n$, (where $b_n = |a_n|$)

(it has successive terms ___________)

Example: $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$

Example: $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n+5}$

Forms for the term that makes the series alternate in sign:

The Alternating Series Test

If the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$ (where $b_n > 0$) satisfies:

$i)$

$ii)$

,then the series is **convergent**.

Note:

$a)$

$b)$ Like the function in the Integral Test, the sequence $\{b_n\}$

Example 1:

$$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n} \qquad b_n =$$

consider $f(x) =$

$f'(x) =$ $\qquad f'(x) < 0$ for all $\qquad \Rightarrow \{b_n\}$ is decreasing

also

$$\underbrace{\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}}\text{ is}$$

Example 2:

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}n^2}{n^2+5} \qquad b_n = \qquad \lim_{n\to\infty}\frac{n^2}{n^2+5}$$

the Alternating Series Test ____________

$$\lim_{n\to\infty}a_n = \lim_{n\to\infty}\frac{(-1)^{n+1}n^2}{n^2+5}$$

The series

Example 3:

$$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\ln n}{n} \qquad b_n =$$

consider $f(x) =$

$f'(x) =$

$f'(x)$ will be negative when

$\{b_n\}$ is decreasing for

$$\lim_{n\to\infty}b_n = \lim_{n\to\infty}\frac{\ln n}{n}$$

$$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\ln n}{n}\text{ is}$$

Alternating Series Estimation Theorem

Spring 2008 # 19

Math 104 – Rimmer
12.5 Alternating Series Test

If we add the first 100 terms of the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\ldots$ how close is the partial sum s_{100} to the sum, s, of the series?

a) $s_{100} > s$ with $s_{100} - s < 1/101$
b) $s_{100} < s$ with $s_{100} - s < 1/101$
c) $s_{100} > s$ with $s_{100} - s < 1/100$
d) $s_{100} < s$ with $s_{100} - s < 1/100$
e) $s_{100} < s$ with $s_{100} - s < 1/e^{101}$
f) cannot be determined