12.5 Alternating Series Test Math 104 - Rimmer 12.5 Alternating Series Test



An _____ is of the form $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (-1)^{n-1} b_n$ or $\sum_{n=1}^{\infty} (-1)^n b_n$, (where (it has successive terms _____) $b_n = |a_n|$

$$b_n = |a_n|$$

Example: $\sum_{n=1}^{\infty} \frac{\left(-1\right)^{n-1}}{n^2}$

Example:
$$\sum_{n=1}^{\infty} \frac{\left(-1\right)^n n^2}{n+5}$$

Forms for the term that makes the series alternate in sign:

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The Alternating Series Test

If the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$ (where $b_n > 0$) satisfies:

- i)
- ii)

,then the series is **convergent**.

Note:

- a)
- b) Like the function in the Integral Test, the sequence $\{b_n\}$

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Example 1:
$$\sum_{n=1}^{\infty} \frac{\left(-1\right)^{n-1}}{n} \qquad b_n$$

consider f(x) =

$$f'(x) =$$

f'(x) = f'(x) < 0 for all $\Rightarrow \{b_n\}$ is decreasing also

$$\sum_{n=1}^{\infty} \frac{\left(-1\right)^{n-1}}{n} \text{ is }$$

$$\sum_{n=1}^{\infty} \frac{\left(-1\right)^{n+1} n^2}{n^2 + 5} \qquad b_n$$

the Alternating Series Test

$$\lim_{n \to \infty} a_n = \lim_{n \to \infty} \frac{(-1)^{n+1} n^2}{n^2 + 5}$$

The series

$$\sum_{n=1}^{\infty} \frac{\left(-1\right)^{n-1} \ln n}{n} \qquad b_n =$$

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 $\operatorname{consider} f(x) =$

$$f'(x) =$$

f'(x) will be negative when

 $\{b_n\}$ is decreasing for

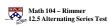
$$\lim_{n\to\infty}b_n=\lim_{n\to\infty}\frac{\ln n}{n}$$

$$\sum_{n=1}^{\infty} \frac{\left(-1\right)^{n-1} \ln n}{n} \text{ is}$$

Alternating Series Estimation Theorem 👼 Math 104 - Rimmer 12.5 Alternating Series Test



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If we add the first 100 terms of the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots$ how

close is the partial sum s_{100} to the sum, s, of the series?

- a) $s_{100} > s$ with $s_{100} s < 1/101$
- b) $s_{100} < s$ with $s_{100} s < 1/101$
- c) $s_{100} > s$ with $s_{100} s < 1/100$
- d) $s_{100} < s$ with $s_{100} s < 1/100$
- e) $s_{100} < s$ with $s_{100} s < 1/e^{101}$
- f) cannot be determined