12.6 Absolute Convergence

An infinite series

$\sum_{n=1}^{\infty} a_n$ is called ____________________ if the positive series $\sum_{n=1}^{\infty} |a_n|$ converges.

__________________ implies ______________.

(If the series of absolute value converges, then the original series also converges)

If the series of absolute value ________, it is still possible for the original series to converge.

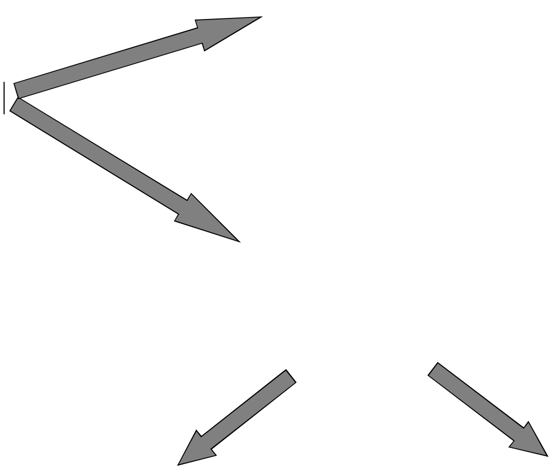
Use the __________________ on the original series.
If the Alternating Series Test gives convergence, then this is a special type of convergence.

An infinite series

$\sum_{n=1}^{\infty} a_n$ is called _____________________ if it converges but $\sum_{n=1}^{\infty} |a_n|$ diverges.

Math 104 – Rimmer
12.6 Absolute Convergence
and the Ratio and Root Tests

$\sum_{n=1}^{\infty} |a_n|$



A major difference between absolutely convergent and conditionally convergent comes in the ______________________.

If $\sum_{n=1}^{\infty} a_n$ is ____________________ with sum s,
then any rearrangement of the sum $\sum_{n=1}^{\infty} a_n$ will ________________.

If $\sum_{n=1}^{\infty} a_n$ is __________________ and r is any real number,
then there is a rearrangement of the sum $\sum_{n=1}^{\infty} a_n$ that ____________.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots = \ln 2$$ (We will show this later)

$$\frac{1}{2}\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots\right) = \frac{1}{2}\ln 2$$

$$\frac{1}{2} - \frac{1}{4} + \frac{1}{6} + \cdots = \frac{1}{2}\ln 2$$

$$0 + \frac{1}{2} + 0 - \frac{1}{4} + 0 + \frac{1}{6} + \cdots = \frac{1}{2}\ln 2$$

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8} + \cdots = \ln 2$$

$$+ \quad 0 + \frac{1}{2} + 0 - \frac{1}{4} + 0 + \frac{1}{6} + 0 + -\frac{1}{8} + \cdots = \frac{1}{2}\ln 2$$

$$1 + \quad \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \quad \frac{1}{7} - \frac{1}{4} + \cdots = \frac{3}{2}\ln 2$$

different sums

same terms

Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

Math 104 – Rimmer
12.6 Absolute Convergence
and the Ratio and Root Tests

$i)\ \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\sqrt{n^3}}$

$ii)\ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$

$iii)\ \sum_{n=1}^{\infty} \frac{(-4)^{n+1}}{3^n}$

12.6 The Ratio Test

Let $\{a_n\}$ be a sequence and assume that the following limit exists: $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L$

$i)$ If $L<1,$ then the series $\sum_{n=1}^{\infty} a_n$ is ________________.

$ii)$ If $L>1$ or if the limit is infinite, then the series $\sum_{n=1}^{\infty} a_n$ is ___________.

$iii)$ If $L=1,$ ________________.
(the series could be absolutely convergent, conditionally convergent, or divergent)

12.6 The Root Test

Let $\{a_n\}$ be a sequence and assume that the following limit exists: $\lim_{n\to\infty}\sqrt[n]{|a_n|} = L$

$i)$ If $L<1,$ then the series $\sum_{n=1}^{\infty} a_n$ is ________________.

$ii)$ If $L>1$ or if the limit is infinite, then the series $\sum_{n=1}^{\infty} a_n$ is ___________.

$iii)$ If $L=1,$ ________________.
(the series could be absolutely convergent, conditionally convergent, or divergent)

Determine whether the series is convergent or divergent.

$i)\ \sum_{n=1}^{\infty} \frac{n^3}{4^n}$ $ii)\ \sum_{n=1}^{\infty} \frac{n!}{4^n}$

$= \infty$

Determine whether the series is convergent or divergent.

Math 104 – Rimmer
12.6 Absolute Convergence
and the Ratio and Root Tests

$iii)\ \sum_{n=1}^{\infty} \frac{(2n+1)^n}{n^{2n}}$ $iv)\ \sum_{n=1}^{\infty} \frac{n^2}{(2n+1)!}$