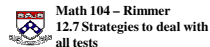
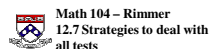


12.7 Strategies (which test to use on which series)




- 1) Check at a glance to see if $\lim_{n \rightarrow \infty} a_n \neq 0$.
If this is true, then the series diverges by the **Test for Divergence**.
- 2) Series that we can find whether or not they converge rather quickly:
 - a) **p -series** $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent for $p > 1$ and divergent if $p \leq 1$.
 - b) **geometric series** $\sum_{n=1}^{\infty} ar^{n-1}$ or $\sum_{n=1}^{\infty} ar^n$ is convergent for $|r| < 1$ and divergent if $|r| \geq 1$.
- 3) Use the Comparison Test / Limit Comparison Test on series
 - a) that have the form of p -series or geometric series
if a_n is a fraction involving polynomials only or polynomials under radicals
compare this series with a p -series
 - b) Note: make sure that the series has only positive terms to use the comp. tests

12.7 Strategies (which test to use on which series)




- 4) The Alternating Series Test might work on series of the form $\sum_{n=1}^{\infty} (-1)^n b_n$
- 5) The Ratio Test works well on series involving factorials
or constants raised to powers involving n
- 6) The Root Test works well if a_n is of the form $(b_n)^n$
- 7) The Integral Test works well if $\int_k^{\infty} f(x) dx$ is not difficult to evaluate,
where $\sum_{n=k}^{\infty} a_n$ with $a_n = f(n)$ and f is continuous, decreasing, and positive on $[k, \infty)$


 Math 104 - Rimmer
 12.7 Strategies to deal with
 all tests


$i) \sum_{n=1}^{\infty} \frac{n}{n^3 + 2}$	$iii) \sum_{n=1}^{\infty} \frac{n^2}{e^n}$	$v) \sum_{n=1}^{\infty} \pi^{1/n}$	$vii) \sum_{n=2}^{\infty} \frac{\ln n}{n^2}$
$ii) \sum_{n=1}^{\infty} \frac{n^2 + 2}{n^3 + 3}$	$iv) \sum_{n=1}^{\infty} \frac{\cos(5n)}{1 + (1.4)^n}$	$vi) \sum_{n=1}^{\infty} \frac{(-1)^{n-1} \sqrt{n}}{n+1}$	$viii) \sum_{n=2}^{\infty} \frac{n \ln n}{(n-1)^3}$

$i) \sum_{n=1}^{\infty} \frac{n}{n^3 + 2}$	$ii) \sum_{n=1}^{\infty} \frac{n^2 + 2}{n^3 + 3}$	$iii) \sum_{n=1}^{\infty} \frac{n^2}{e^n}$
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 Math 104 - Rimmer
 12.7 Strategies to deal with
 all tests


$i) \sum_{n=1}^{\infty} \frac{n}{n^3 + 2}$	$iii) \sum_{n=1}^{\infty} \frac{n^2}{e^n}$	$v) \sum_{n=1}^{\infty} \pi^{1/n}$	$vii) \sum_{n=2}^{\infty} \frac{\ln n}{n^2}$
$ii) \sum_{n=1}^{\infty} \frac{n^2 + 2}{n^3 + 3}$	$iv) \sum_{n=1}^{\infty} \frac{\cos(5n)}{1 + (1.4)^n}$	$vi) \sum_{n=1}^{\infty} \frac{(-1)^{n-1} \sqrt{n}}{n+1}$	$viii) \sum_{n=2}^{\infty} \frac{n \ln n}{(n-1)^3}$

$iv) \sum_{n=1}^{\infty} \frac{\cos(5n)}{1 + (1.4)^n}$	$v) \sum_{n=1}^{\infty} \pi^{1/n}$	$vi) \sum_{n=1}^{\infty} \frac{(-1)^{n-1} \sqrt{n}}{n+1}$
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 Math 104 – Rimmer
 12.7 Strategies to deal with
 all tests

i) $\sum_{n=1}^{\infty} \frac{n}{n^3 + 2}$	iii) $\sum_{n=1}^{\infty} \frac{n^2}{e^n}$	v) $\sum_{n=1}^{\infty} \pi^{1/n}$	vii) $\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$
ii) $\sum_{n=1}^{\infty} \frac{n^2 + 2}{n^3 + 3}$	iv) $\sum_{n=1}^{\infty} \frac{\cos(5n)}{1 + (1.4)^n}$	vi) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \sqrt{n}}{n+1}$	viii) $\sum_{n=2}^{\infty} \frac{n \ln n}{(n-1)^3}$

vii) $\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$	viii) $\sum_{n=2}^{\infty} \frac{n \ln n}{(n-1)^3}$ $b_n = \frac{n \ln n}{n^3} = \frac{\ln n}{n^2}$
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 12.7 Strategies to deal with
 all tests

12.2

Test for Divergence :

If $\lim_{n \rightarrow \infty} a_n \neq 0$ or $\lim_{n \rightarrow \infty} a_n$ does not exist, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

geometric series $\sum_{n=1}^{\infty} ar^{n-1}$ or $\sum_{n=1}^{\infty} ar^n$ is convergent for $|r| < 1$ and
 divergent if $|r| \geq 1$.

12.3 The Integral Test

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If $f(x)$ is: a) continuous, on the interval $[k, \infty)$
b) positive, constant $k > 0$
c) and decreasing

, then the series $\sum_{n=k}^{\infty} a_n$ (with $a_n = f(n)$)

i) is convergent when $\int_k^{\infty} f(x) dx$ is convergent.

ii) is divergent when $\int_k^{\infty} f(x) dx$ is divergent.

p -series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent for $p > 1$ and divergent if $p \leq 1$.

12.4

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all tests

The Comparison Test:

Given the series $\sum_{n=1}^{\infty} a_n$, ($a_n \geq 0$)

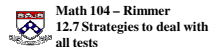
(i) if the terms a_n are **smaller** than the terms b_n of a known **convergent**

series $\sum_{n=1}^{\infty} b_n$ ($b_n \geq 0$), then our series $\sum_{n=1}^{\infty} a_n$ is also **convergent**.

(ii) if the terms a_n are **larger** than the terms b_n of a known **divergent**

series $\sum_{n=1}^{\infty} b_n$ ($b_n \geq 0$), then our series $\sum_{n=1}^{\infty} a_n$ is also **divergent**.

12.4



The Limit Comparison Test:

Given the series $\sum_{n=1}^{\infty} a_n$, ($a_n > 0$) and a

known convergent or divergent series $\sum_{n=1}^{\infty} b_n$, ($b_n > 0$)

If the $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c$ where c is a finite positive number, then

the series will behave alike, i.e. either both converge or both diverge.

12.5 The Alternating Series Test

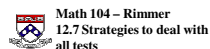
If the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$ (where $b_n > 0$) satisfies:

i) $\lim_{n \rightarrow \infty} b_n = 0$

ii) $\{b_n\}$ is a decreasing sequence, and

,then the series is **convergent**.

12.6 The Ratio Test



Let $\{a_n\}$ be a sequence and assume that the following limit exists: $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L$

i) If $L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.

ii) If $L > 1$ or if the limit is infinite, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

iii) If $L = 1$, the Ratio Test is inconclusive.

(the series could be absolutely convergent, conditionally convergent, or divergent)

12.6 The Root Test



Let $\{a_n\}$ be a sequence and assume that the following limit exists: $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = L$

- i) If $L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.
- ii) If $L > 1$ or if the limit is infinite, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.
- iii) If $L = 1$, the Root Test is inconclusive.
(the series could be absolutely convergent, conditionally convergent, or divergent)