12.7 Strategies (which test to use on which series)

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12.7 Strategies to deal with all tests

1) Check at a glance to see if $\lim_{n\to\infty} a_n \neq 0$.

If this is true, then the series diverges by the **Test for Divergence**.

2) Series that we can find whether or not they converge rather quickly:

$a)$ $\boxed{p-\text{series}}$ $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent for $p > 1$ and divergent if $p \leq 1$.

$b)$ $\boxed{\text{geometric series}}$ $\sum_{n=1}^{\infty} ar^{n-1}$ or $\sum_{n=1}^{\infty} ar^{n}$ is convergent for $|r| < 1$ and divergent if $|r| \geq 1$.

3) Use the Comparison Test / Limit Comparison Test on series

$a)$ that have the form of $p-$series or geometric series

if a_n is a fraction involving polynomials only or polynomials under radicals compare this series with a $p-$series

$b)$ Note: make sure that the series has only positive terms to use the comp. tests

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4) The Alternating Series Test might work on series of the form $\sum_{n=1}^{\infty} (-1)^n b_n$

5) The Ratio Test works well on series involving factorials or constants raised to powers involving n

6) The Root Test works well if a_n is of the form $(b_n)^n$

7) The Integral Test works well if $\int_k^{\infty} f(x)\,dx$ is not difficult to evaluate,

where $\sum_{n=k}^{\infty} a_n$ with $a_n = f(n)$ and f is continuous, decreasing, and positive on $[k, \infty)$

$i)\ \sum_{n=1}^{\infty}\frac{n}{n^3+2}$ $iii)\ \sum_{n=1}^{\infty}\frac{n^2}{e^n}$ $v)\ \sum_{n=1}^{\infty}\pi^{1/n}$ $vii)\ \sum_{n=2}^{\infty}\frac{\ln n}{n^2}$

$ii)\ \sum_{n=1}^{\infty}\frac{n^2+2}{n^3+3}$ $iv)\ \sum_{n=1}^{\infty}\frac{\cos(5n)}{1+(1.4)^n}$ $vi)\ \sum_{n=1}^{\infty}\frac{(-1)^{n-1}\sqrt{n}}{n+1}$ $viii)\ \sum_{n=2}^{\infty}\frac{n\ln n}{(n-1)^3}$

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$vii)\ \sum_{n=2}^{\infty}\frac{\ln n}{n^2}$

$viii)\ \sum_{n=2}^{\infty}\frac{n\ln n}{(n-1)^3}$ $\quad b_n=\frac{n\ln n}{n^3}=\frac{\ln n}{n^2}$

12.2

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Test for Divergence :

If $\lim_{n\to\infty}a_n\neq 0$ or $\lim_{n\to\infty}a_n$ does not exist, then the series $\sum_{n=1}^{\infty}a_n$ is divergent.

geometric series $\sum_{n=1}^{\infty}ar^{n-1}$ or $\sum_{n=1}^{\infty}ar^n$ is convergent for $|r|<1$ and divergent if $|r|\geq 1$.

12.3 The Integral Test

If $f(x)$ is: $a)$ continuous, on the interval $[k,\infty)$ constant $k>0$

$b)$ positive,

$c)$ and decreasing

,then the series $\sum_{n=k}^{\infty} a_n$ $\left(\text{with } a_n = f(n)\right)$

$i)$ is convergent when $\int_k^{\infty} f(x)\,dx$ is convergent.

$ii)$ is divergent when $\int_k^{\infty} f(x)\,dx$ is divergent.

$\boxed{p-\text{series}}$ $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent for $p>1$ and divergent if $p \le 1$.

12.4

The Comparison Test:

Given the series $\sum_{n=1}^{\infty} a_n$, $(a_n \ge 0)$

(i) if the terms a_n are **smaller** than the terms b_n of a known **convergent** series $\sum_{n=1}^{\infty} b_n$ $(b_n \ge 0)$, then our series $\sum_{n=1}^{\infty} a_n$ is also **convergent**.

(ii) if the terms a_n are **larger** than the terms b_n of a known **divergent** series $\sum_{n=1}^{\infty} b_n$ $(b_n \ge 0)$, then our series $\sum_{n=1}^{\infty} a_n$ is also **divergent**.

12.4

The Limit Comparison Test:

Given the series $\sum_{n=1}^{\infty} a_n$, $(a_n > 0)$ and a

known convergent or divergent series $\sum_{n=1}^{\infty} b_n$, $(b_n > 0)$

If the $\lim_{n\to\infty} \frac{a_n}{b_n} = c$ where c is a finite positive number, then

the series will behave alike, i.e. either both converge or both diverge.

12.5 The Alternating Series Test

If the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$ $(\text{where } b_n > 0)$ satisfies:

$i)$ $\lim_{n\to\infty} b_n = 0$

$ii)$ $\{b_n\}$ is a decreasing sequence, and

,then the series is **convergent**.

12.6 The Ratio Test

Let $\{a_n\}$ be a sequence and assume that the following limit exists: $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L$

$i)$ If $L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.

$ii)$ If $L > 1$ or if the limit is infinite, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

$iii)$ If $L = 1$, the Ratio Test is inconclusive.

(the series could be absolutely convergent, conditionally convergent, or divergent)

12.6 The Root Test

Let $\{a_n\}$ be a sequence and assume that the following limit exists: $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L$

$i)$ If $L < 1,$ then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.

$ii)$ If $L > 1$ or if the limit is infinite, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

$iii)$ If $L = 1,$ the Root Test is inconclusive.
(the series could be absolutely convergent, conditionally convergent, or divergent)