

## 12.8 Power Series

A power series is a series of the form


$$\sum_{n=0}^{\infty} c_n x^n =$$

where:

- a)
- b)

For each fixed  $x$ , the series above is a series of constants that we can test for convergence or divergence.

A power series may converge for some values of  $x$  and diverge for other values of  $x$ .

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The sum of the series is a function

whose \_\_\_\_\_ is the set of all  $x$  for which the series converges.

$f(x)$  is reminiscent of a \_\_\_\_\_ but it has infinitely many terms

If all  $c_n$ 's = 1, we have

$$f(x) = 1 + x + x^2 + \dots + x^n + \dots = \sum_{n=0}^{\infty} x^n$$

This is the \_\_\_\_\_ with \_\_\_\_\_.

The power series will converge for \_\_\_\_\_ and diverge for all other  $x$ .

In general, a series of the form

is called a power series \_\_\_\_\_ or a power series about  $a$

We use the \_\_\_\_\_ to find for what values of  $x$  the series converges.

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L \text{ _____}$$

solve for  $|x - a|$  to get  $|x - a| < R$

$$\Rightarrow -R < x - a < R$$

$$\Rightarrow a - R < x < a + R$$

$R$  is called the \_\_\_\_\_  
\_\_\_\_\_ (R.O.C.).

This is called the \_\_\_\_\_ Plug in the endpoints to check for convergence  
\_\_\_\_\_ (I.O.C.). or divergence at the endpoints.

Find the radius of convergence and the interval of convergence.

$$\sum_{n=1}^{\infty} \frac{(-1)^n n^2 x^n}{2^n}$$


$x =$  \_\_\_\_\_

$x =$  \_\_\_\_\_

Radius of convergence: \_\_\_\_\_

Interval of convergence: \_\_\_\_\_

Find the radius of convergence and the interval of convergence.

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$$\sum_{n=1}^{\infty} \frac{3^n (x+4)^n}{\sqrt{n}}$$


$x =$  \_\_\_\_\_

$x =$  \_\_\_\_\_

R.O.C.: \_\_\_\_\_

I.O.C.: \_\_\_\_\_

Find the radius of convergence and the interval of convergence.

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$$\sum_{n=1}^{\infty} \frac{(4x+1)^n}{n^2}$$

Check endpoints:

$x =$  \_\_\_\_\_

$x =$  \_\_\_\_\_

R.O.C.: \_\_\_\_\_

I.O.C.: \_\_\_\_\_

Sometimes the Root Test can be used just as the Ratio Test.

When  $a_n$  can be written as  $(b_n)^n$ , then the Root Test should be used.

$$\sum_{n=1}^{\infty} \frac{3^n (x-5)^n}{n^n}$$

R.O.C. = _____
I.O.C. = _____

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$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = 0 \Rightarrow$$

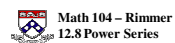
$$\sum_{n=1}^{\infty} \frac{n!(x-7)^n}{2^n}$$

R.O.C. = _____
I.O.C. = _____

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$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \infty \Rightarrow$$

Find the radius of convergence.



$$\sum_{n=1}^{\infty} \frac{(-1)^n (n!)^2 x^{2n}}{(2n)!}$$

Radius of convergence: