12.8 Power Series

A power series is a series of the form

$$\sum_{n=0}^{\infty} c_n x^n =$$

where:

$a)$

$b)$

For each fixed x, the series above is a series of constants that we can test for convergence or divergence.

A power series may converge for some values of x and diverge for other values of x.

The sum of the series is a function

whose _________ is the set of all x for which the series converges.

$f(x)$ is reminiscent of a __________ but it has infinitely many terms

If all c_n's $= 1$, we have

$$f(x) = 1 + x + x^2 + \ldots + x^n + \ldots = \sum_{n=0}^{\infty} x^n$$

This is the _______________ with _____.

The power series will converge for ____ and diverge for all other x.

In general, a series of the form

is called a power series __________ or a power series about a

We use the ________________________ to find for what values of x the series converges.

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L$ _______________

R is called the ________ ____________ (R.O.C.).

solve for $|x-a|$ to get $|x-a| < R$

$\Rightarrow -R < x - a < R$

$\Rightarrow a - R < x < a + R$

This is called the _______ ______________ (I.O.C.).

Plug in the endpoints to check for convergence or divergence at the endpoints.

Find the radius of convergence and the interval of convergence.

$$\sum_{n=1}^{\infty}\frac{(-1)^n n^2 x^n}{2^n}$$

$x =$ ___　　　　$x =$ ___

Radius of convergence: ______
Interval of convergence: _______

Find the radius of convergence and the interval of convergence.

$$\sum_{n=1}^{\infty} \frac{3^n (x+4)^n}{\sqrt{n}}$$

$x =$ ___ $x =$ ___

R.O.C.: _______

I.O.C. : _______

Find the radius of convergence and the interval of convergence.

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$$\sum_{n=1}^{\infty} \frac{(4x+1)^n}{n^2}$$

Check endpoints:

$x =$ ___ $x =$ ___

R.O.C.: ______

I.O.C. : ______

Sometimes the Root Test can be used just as the Ratio Test.

When a_n can be written as $(b_n)^n$, then the Root Test should be used.

$$\sum_{n=1}^{\infty} \frac{3^n (x-5)^n}{n^n}$$

R.O.C. = _____
I.O.C. = _____

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = 0 \Rightarrow$$

$$\sum_{n=1}^{\infty} \frac{n!(x-7)^n}{2^n}$$

R.O.C. = ____
I.O.C. = ____

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \infty \Rightarrow$$

Find the radius of convergence.

$$\sum_{n=1}^{\infty} \frac{(-1)^n (n!)^2 x^{2n}}{(2n)!}$$

Radius of convergence: ____