12.9 Functions as Power Series

The very first function we have seen represented as a power series is the geometric series with $a = 1$ and $r = x$

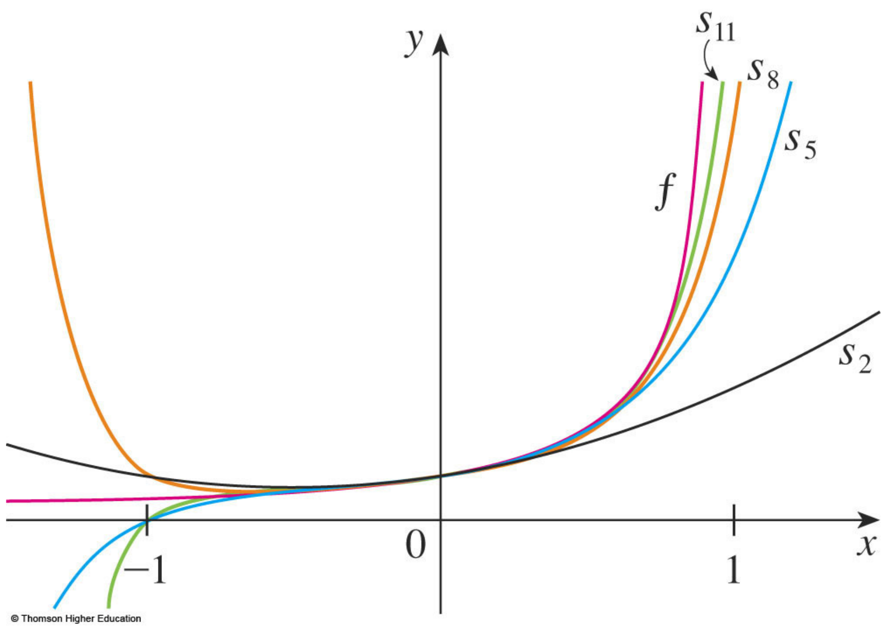
$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots \quad , |x| < 1$$

We can find the power series representation of other functions by algebraically manipulating them to to be some multiple of this series.

$$\frac{1}{1+x} = \frac{1}{1-(-x)}$$

The interval of convergence remains unchanged since this is still a type of geometric series.

$$\frac{1}{1+x} =$$

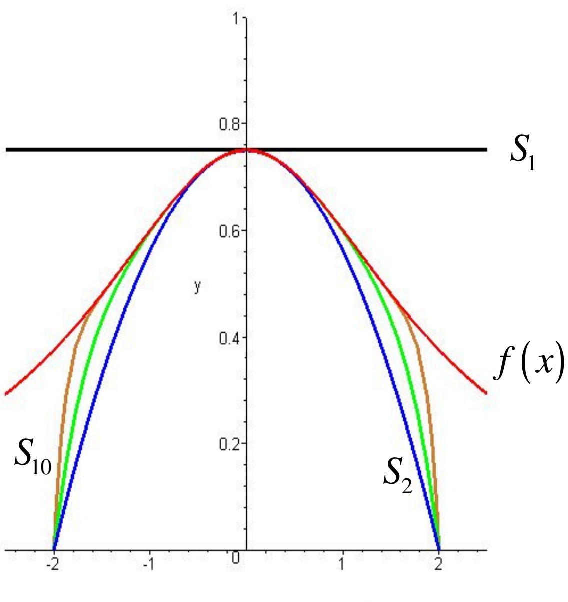


Represent the function as a power series and determine the interval of convergence.

$$\frac{3}{4+x^2}$$

, so the interval of convergence is

$$\frac{3}{4+x^2} =$$



$$f(x)=\sum_{n=0}^{\infty}c_n(x-a)^n=c_0+c_1(x-a)+c_2(x-a)^2+c_3(x-a)^3+\cdots$$

If the power series representation of $f(x)$ has a radius of convergence $R>0$,

we can obtain a power series representation for $f'(x)$ by

term - by - term ___________:

$$f(x)=c_0+c_1(x-a)+c_2(x-a)^2+c_3(x-a)^3+\cdots$$

$$f'(x)=c_1+2c_2(x-a)+3c_3(x-a)^2+\cdots$$

$$f'(x)=\frac{d}{dx}\left(\sum_{n=0}^{\infty}c_n(x-a)^n\right)=\sum_{n=0}^{\infty}\frac{d}{dx}\left[c_n(x-a)^n\right]=\sum_{n=1}^{\infty}nc_n(x-a)^{n-1}$$

with the same radius of convergence R

starts at $n=1$

we can obtain a power series representation for $\int f(x)\,dx$ by

term - by - term __________:

$$\int f(x)\,dx=C+c_0x+c_1\frac{(x-a)^2}{2}+c_2\frac{(x-a)^3}{3}+c_3\frac{(x-a)^4}{4}+\cdots$$

$$\int\left(\sum_{n=0}^{\infty}c_n(x-a)^n\right)dx=\sum_{n=0}^{\infty}\int\left[c_n(x-a)^n\right]dx=C+\sum_{n=0}^{\infty}\frac{c_n(x-a)^{n+1}}{n+1}$$

with the same radius of convergence R

C is a constant of integration

Represent the function as a power series and determine the radius of convergence.

$$f(x)=\frac{x^3}{(1-x)^2}$$

$$g(x)=\frac{1}{1-x}=\sum_{n=0}^{\infty}x^n \quad \text{with } R=1$$

Represent the function as a power series and determine the radius of convergence.

$f(x) = \arctan x$

$$\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n}, \quad \text{with } R = 1$$

$$= 1 - x^2 + x^4 - x^6 + \ldots$$

$\arctan x = C +$

$\arctan x =$

Represent the function as a power series and determine the radius of convergence.

$f(x) = \ln(1-x)$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad \text{with } R = 1$$

$-\ln(1-x) =$

$\ln(1-x) =$

$\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$,with $R = 1$

$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$

$\arctan 1 =$

$\ln(1-x) = -\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1}$,with $R = 1$

$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} + \ldots$

$\ln\left(1-\frac{1}{2}\right) =$

$\ln\left(\frac{1}{2}\right) =$

$\ln 1 - \ln 2 =$

Algebraically manipulate $\frac{1}{(1-x)^2}$ (the same way we manipulated $\frac{1}{1-x}$)

$\frac{1}{(1-x)^2} = \sum_{n=1}^{\infty} nx^{n-1}$ with $R = 1$

Represent $\frac{1}{(4-3x)^2}$ as a power series and determine the radius of convergence.

$\frac{1}{(4-3x)^2}$

Algebraically manipulate $\ln(1-x)$ $\left(\text{the same way we manipulated } \frac{1}{1-x}\right)$

$$\ln(1-x) = -\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1} \text{ ,with } R = 1$$

Represent $\ln(3+2x)$ as a power series and determine the radius of convergence.

$\ln(3+2x)$