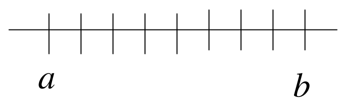
Average of n numbers:

$$\frac{x_1 + x_2 + x_3 + \cdots x_n}{n}$$

Take the n numbers to be sample points for a function:

$$\frac{f\left(x_1^*\right) + f\left(x_2^*\right) + f\left(x_3^*\right) + \cdots f\left(x_n^*\right)}{n}$$

Partition the interval $[a,b]$ into n subintervals of equal length.



Here $n = 8$.

What is the width of each subinterval?

Call this

$\Rightarrow n =$

We now have:

$$\frac{f\left(x_1^*\right) + f\left(x_2^*\right) + f\left(x_3^*\right) + \cdots f\left(x_n^*\right)}{n} = \underline{\qquad\qquad}$$

Math 104 – Rimmer
6.5 Average Value of a Function

Taking the limit as $n \to \infty$, we get $\dfrac{\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} f\left(x_i^*\right)\Delta x}{b-a}$

Average value of $f(x)$ on an interval $[a,b]$

Find the average value of the function $\dfrac{3}{(1+r)^2}$ on the interval $[1,6]$.

Mean Value Theorem for Integrals

Let f be ________ on $[a,b]$,

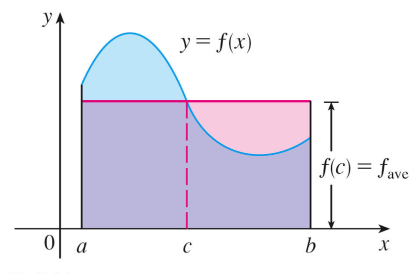
then there is a value c in $[a,b]$ such that

$$f(c) =$$

or

$$f(c)(b-a) = \int_a^b f(x)\,dx$$

$$=$$



Proof:

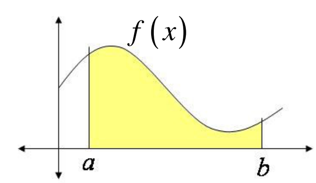
Let f be continuous on $[a,b]$,

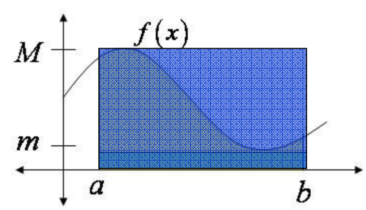
By the ______ Value Theorem,

there is a m and M such that

Then $\qquad \le \int_a^b f(x)\,dx \le$

$\Rightarrow$





By the _________ Value Theorem,

there is a c in $[a,b]$ with $f(c) = \dfrac{1}{b-a}\int_a^b f(x)\,dx.$

(a) Find the average value of $f(x)=\sqrt{x}$ on the interval $[0,4]$.

(b) Find c such that $f_{ave}=f(c)$.

(c) Sketch the graph of f and a rectangle whose area is the same as the area under the graph of f.