Section 8.1 Integration By Parts

Goal : To be able to integrate ______________.

Chapter 8 : Techniques of Integration

8.1 :

8.2 :

8.3 :

8.4 :

Integration using __________ can be thought of as the __________ in reverse.

Integration by parts can be thought of as the __________ in reverse.

$$\frac{d}{dx}\left[f(x)\cdot g(x)\right]=$$

$$\int \frac{d}{dx}\left[f(x)\cdot g(x)\right]dx=$$

$$=\int\left[f'(x)\cdot g(x)\right]dx+\int\left[f(x)\cdot g'(x)\right]dx$$

$$f(x)\cdot g(x)-\int\left[f'(x)\cdot g(x)\right]dx=$$

$$\int\left[f(x)\cdot g'(x)\right]dx=$$

$$\int \left[f(x) \cdot g'(x) \right] dx = f(x) \cdot g(x) - \int \left[f'(x) \cdot g(x) \right] dx$$

$u = f(x) \qquad v = g(x)$

$du = f'(x)\,dx \qquad dv = g'(x)\,dx$

$$\int u\,dv =$$

Choose u and choose dv

Big Picture: We are trading in _________ for _______

Goal: To get a ______ integral than the original one

1. Choose u to be a function that becomes simpler when ___________
2. Make sure dv can be readily ________

$$\int x^2 e^{5x}\,dx$$

Wrong Way

$u = \qquad dv =$

$du = \qquad v =$

traded in this — for this

$$\int x^2 e^{5x}\,dx =$$

Correct Way

$u = \qquad dv =$

$du = \qquad v =$

traded in this — for this

$$\int x^2 e^{5x}\,dx =$$

$$\int x^2 e^{5x}\,dx = \tfrac{1}{5} x^2 e^{5x} - \tfrac{2}{5} \boxed{\int x e^{5x}\,dx}$$

I.B.P. again

$u = \qquad dv =$

$du = \qquad v =$

$$\int x^2 e^{5x}\,dx = \tfrac{1}{5} x^2 e^{5x} - \tfrac{2}{5}$$

$$\int x^2 e^{5x}\,dx = \tfrac{1}{5} x^2 e^{5x} -$$

$$\boxed{\int x^2 e^{5x}\,dx = \tfrac{1}{5} x^2 e^{5x} - \tfrac{2}{25} x e^{5x} + \tfrac{2}{125} e^{5x} + C}$$

Shortcut: Works when you have one of the following two situations :

1. $\int(\text{polynomial})(\text{exponential})\,dx$
2. $\int(\text{polynomial})(\text{trig.})\,dx$

$\int x^2 e^{5x}\,dx$

Step 1: Differentiate the polynomial down to 0.

Step 2: Integrate the trig. or exponential the same amount of times

Diff *Int*

Step 3: Multiply along diagonals going down to the right applying an alternating sign starting with +

$$\int x^2 e^{5x}\,dx = \tfrac{1}{5}x^2 e^{5x} - \tfrac{2}{25}xe^{5x} + \tfrac{2}{125}e^{5x} + C$$

$\int_0^1 x\cos(\pi x)\,dx$

Diff *Int*

$\int_0^1 x\cos(\pi x)\,dx =$

$=$

$=$

$$\int_4^9 \frac{\ln y}{\sqrt{y}}\,dy$$

✖ **Short-cut doesn't work here**

Wrong Way

$u =$ $dv =$

$du =$ $v =$

$f = y^{-1/2}$

$f' =$ $\Rightarrow f' =$

Correct Way

$u =$ $dv =$

$du =$ $v =$

$\int y^{-1/2}dy =$

$$\int \frac{\ln y}{\sqrt{y}}\,dy = 2y^{1/2}\ln y - \int \frac{2y^{1/2}}{y}\,dy$$

$$\int_4^9 \frac{\ln y}{\sqrt{y}}\,dy = \Big[2\sqrt{y}\ln y - 4\sqrt{y}\Big]_4^9$$

Math 104 – Rimmer
8.1 Integration By Parts

$$\int_1^{\sqrt{3}} \arctan\left(\tfrac{1}{x}\right)dx$$

✖ **Short-cut doesn't work here**

$f = \arctan\left(\tfrac{1}{x}\right)$

Only Way

$u =$ $dv =$

$f' =$

$du =$ $v =$

$$\int \arctan\left(\tfrac{1}{x}\right)dx = x\arctan\left(\tfrac{1}{x}\right) + \int \frac{x}{x^2+1}\,dx$$

$$\int \arctan\left(\tfrac{1}{x}\right)dx = x\arctan\left(\tfrac{1}{x}\right) + \tfrac{1}{2}\ln\left(x^2+1\right)$$

$$\int_1^{\sqrt{3}} \arctan\left(\tfrac{1}{x}\right)dx = \left[x\arctan\left(\tfrac{1}{x}\right) + \tfrac{1}{2}\ln\left(x^2+1\right)\right]_1^{\sqrt{3}}$$

$\int e^{-t}\sin(t)\,dt$ ✖ **Short-cut doesn't work here**

$u=\sin(t) \quad dv=e^{-t}dt$

$du=\cos(t)\,dt \quad v=-e^{-t}$

$$\Rightarrow \int e^{-t}\sin(t)\,dt=-e^{-t}\sin(t)+\int e^{-t}\cos(t)\,dt$$

$u=\cos(t) \quad dv=e^{-t}dt$

$du=-\sin(t)\,dt \quad v=-e^{-t}$

$$\Rightarrow \int e^{-t}\sin(t)\,dt=-e^{-t}\sin(t)+\underbrace{\int e^{-t}\cos(t)\,dt}$$

$$-e^{-t}\cos(t)-\int e^{-t}\sin(t)\,dt$$

$$\Rightarrow \int e^{-t}\sin(t)\,dt=-e^{-t}\sin(t)-e^{-t}\cos(t)-\int e^{-t}\sin(t)\,dt$$

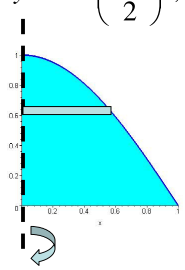
$$+\int e^{-t}\sin(t)\,dt \qquad\qquad +\int e^{-t}\sin(t)\,dt$$

$$2\int e^{-t}\sin(t)\,dt=-e^{-t}\sin(t)-e^{-t}\cos(t)$$

$$\boxed{\int e^{-t}\sin(t)\,dt=\frac{1}{2}\left[-e^{-t}\sin(t)-e^{-t}\cos(t)\right]}$$

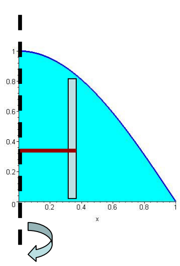
Find the volume of the solid of revolution formed by rotating the region bounded by the curves

$y=\cos\left(\frac{\pi x}{2}\right)$, $y=0,\ 0\le x\le 1$ about the $y-$axis.



<u>Try disk method</u>

Problem :



<u>Use Shells</u>

radius :

height :

$$V=\int_a^b 2\pi(\text{radius})(\text{height})\,dx$$

$$V = 2\pi\int_0^1 x\cos\left(\frac{\pi x}{2}\right)dx$$

<u>*Diff*</u> <u>*Int*</u>

$$V = 2\pi\left[\frac{2x}{\pi}\sin\left(\frac{\pi x}{2}\right)+\frac{4}{\pi^2}\cos\left(\frac{\pi x}{2}\right)\right]_0^1$$

$$V = 2\pi\left(\qquad\qquad\qquad\qquad \right)$$