8.2 Integrating Powers of Trig. Functions

$\int \cos^m x \sin^p x dx \quad (m > 0, p > 0)$

If the power of $\cos x$ is **odd** and the power of $\sin x$ is **even**:

$\overbrace{m = 2k+1}$

"peel off" one power of cosine

$\cos^m x = [\cos x]^m = [\cos x]^{2k+1} = [\cos x]^{2k} \cos x$

$\int [\cos x]^{2k} \cos x \sin^p x dx = \int [\cos x]^{2k} \sin^p x \cos x dx$

now cosine is raised to an even power, we will now convert $[\cos x]^{2k}$

into sines using $\boxed{\cos^2 x = 1 - \sin^2 x}$ $\quad [\cos x]^{2k} = \left[(\cos x)^2\right]^k = (1 - \sin^2 x)^k$

$= \int \left[1 - \sin^2 x\right]^k \sin^p x \cos x dx = \int \left[1 - u^2\right]^k u^p du$ multiply out and integrate the polynomial in u

finally we use u-substitution with $\boxed{u = \sin x}$

the $\cos x$ that you peeled off will be used for du

$\int \cos^5 x \sin^2 x dx$

$$\int \sin^3 x dx$$

"peel off" one power of sine

convert into cosines using $\boxed{\sin^2 x = 1 - \cos^2 x}$

use u-substitution with $\boxed{u = \cos x}$

$$\int_0^{\pi/4} \cos^5 x \sin^3 x dx$$

if both powers are odd, "peel off" one power of either and do nothing to the other

$$\int_0^{\pi} \cos^4 x \sin^2 x dx$$

if both powers are even, then use half angle identites on both.

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x) \qquad \cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

Find the volume of the solid obtained by rotating the region bounded by $y = \sin^2 x$ and $y = 0$ for $0 \le x \le \pi$ about the x-axis.