Section 8.4 : Partial Fraction Decomposition

Rational Function :

The ______ of a polynomial is the highest exponent on x

$\left.\begin{matrix} x-4 \\ 2x+3 \end{matrix}\right\}$ $\left.\begin{matrix} 2x^2-5x-12 \\ x^2-x+3 \end{matrix}\right\}$

$2x^2-5x-12$ is called a ________ quadratic polynomial since it can be factored (over the reals)

$2x^2-5x-12=$ the roots are ____________, $b^2-4ac\geq 0$

x^2-x+3 is called a _________ quadratic polynomial since it cannot be factored (over the reals)

the roots are ______________, $b^2-4ac<0$

Every polynomial of degree $n>0$ with real coefficients can be written as a product of __________________________ factors.

Goal : To integrate ______________

- Write $q(x)$ in this manner and then express the rational function as ______________________
- The simpler fractions should be _____________.

simpler fractions : $\dfrac{1}{x-4}$ $\dfrac{1}{(x-4)^2}$ $\dfrac{1}{x^2+4}$

that can be integrated :

$$\int \frac{1}{x-4}\,dx=$$

$$\int \frac{1}{(x-4)^2}\,dx=$$

$$\int \frac{1}{x^2+4}\,dx=$$

Partial Fraction Decomposition: $\frac{p(x)}{q(x)}$

1. The degree of the denominator ____ be greater than the degree of the numerator
   If it is not, then __________ the denominator into the numerator.

2. Decompose the fraction in the following manner: $(A, B, C,$ and D are constants$)$

   $i)$ $q(x)$ can be written as a product of only linear polynoimials

   $$\frac{5x}{(x-4)(2x+3)}$$

   $ii)$ $q(x)$ can be written as a product involving powers of linear polynoimials

   $$\frac{x^2+6x-4}{(x-3)^3(x+5)}$$

   $iii)$ $q(x)$ can be written as a product involving irreducible quadratic polynoimials

   $$\frac{16x-5}{(x^2+10x+2)(x-7)}$$

3. Use the method of _______________ to find the constants $A, B, C,$ and D

$$\int_3^4 \frac{4}{x^2-4}\,dx$$

$$\int \frac{x+8}{x(x+2)^2}\,dx$$

Focus only on the numerator:

This equation should be true for all x, so choose three different values of x:

Plug A in and work with this equation:

$$\int \frac{x+8}{x(x+2)^2}\,dx \text{ continued:}$$

$$\int_2^3 \frac{x^3 - x^2 - 1}{x^2 - x}\,dx$$

$$\int \frac{3-x}{x\left(x^2+1\right)}dx$$

Match up coefficients from the left and right hand sides