## **Section 8.4 : Partial Fraction Decomposition**



## **Rational Function**:

The \_\_\_\_\_ of a polynomial is the highest exponent on x

$$\begin{array}{c}
 x-4 \\
 2x+3
 \end{array}$$

$$\begin{array}{c}
 2x^2-5x-12 \\
 x^2-x+3
 \end{array}$$

 $2x^2-5x-12$  is called a \_\_\_\_\_ quadratic polynomial since it can be factored (over the reals)

$$2x^2 - 5x - 12 =$$
 the roots are \_\_\_\_\_\_,  $b^2 - 4ac \ge 0$ 

 $x^2-x+3$  is called a \_\_\_\_\_ quadratic polynomial since it cannot be factored (over the reals) the roots are \_\_\_\_\_,  $b^2-4ac<0$ 

Every polynomial of degree n > 0 with real coefficients can be written as a product of \_\_\_\_\_\_ factors.

Goal: To integrate \_\_\_\_\_

- Write q(x) in this manner and then express the rational function as \_\_\_\_\_
- The simpler fractions should be \_\_\_\_\_\_

simpler fractions:  $\frac{1}{x-4}$   $\frac{1}{(x-4)^2}$   $\frac{1}{x^2+4}$ 

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that can be integrated :

$$\int \frac{1}{x-4} \, dx =$$

$$\int \frac{1}{\left(x-4\right)^2} dx =$$

$$\int \frac{1}{x^2 + 4} \, dx =$$

## **Partial Fraction Decomposition:**





- 1. The degree of the denominator \_\_\_\_\_ be greater than the degree of the numerator If it is not, then \_\_\_\_\_ the denominator into the numerator.
- 2. Decompose the fraction in the following manner: (A,B,C, and D are constants)
  - i) q(x) can be written as a product of only linear polynomials

$$\frac{5x}{(x-4)(2x+3)}$$

ii) q(x) can be written as a product involving powers of linear polynoimials

$$\frac{x^2 + 6x - 4}{(x - 3)^3 (x + 5)}$$

iii) q(x) can be written as a product involving irreducible quadratic polynoimials

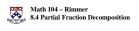
$$\frac{16x-5}{(x^2+10x+2)(x-7)}$$

3. Use the method of \_\_\_\_\_\_ to find the constants A, B, C, and D

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 $\int_{3}^{4} \frac{4}{x^2 - 4} dx$ 

$$\int \frac{x+8}{x(x+2)^2} dx$$

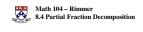


Focus only on the numerator:

This equation should be true for all x, so choose three different values of x:

Plug A in and work with this equation:

$$\int \frac{x+8}{x(x+2)^2} dx$$
 continued:



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$$\int_{2}^{3} \frac{x^3 - x^2 - 1}{x^2 - x} dx$$

$$\int \frac{3-x}{x(x^2+1)} dx$$

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Match up coefficients from the left and right hand sides