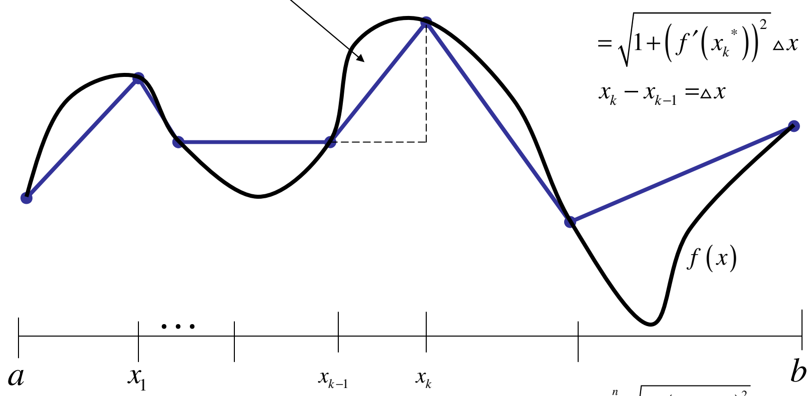
$$\sqrt{\left(x_k - x_{k-1}\right)^2 + \left(f\left(x_k\right) - f\left(x_{k-1}\right)\right)^2}$$

$$= \sqrt{\left(x_k - x_{k-1}\right)^2 + \left(f'\left(x_k^*\right)\left(x_k - x_{k-1}\right)\right)^2}$$

$$= \sqrt{1 + \left(f'\left(x_k^*\right)\right)^2}\,\Delta x$$

$$x_k - x_{k-1} = \Delta x$$



by the Mean Value Theorem: $\exists\, x_k^* \in \left(x_{k-1}, x_k\right)$ such that

$$\frac{f\left(x_k\right) - f\left(x_{k-1}\right)}{x_k - x_{k-1}} = f'\left(x_k^*\right)$$

$$\Rightarrow f\left(x_k\right) - f\left(x_{k-1}\right) = f'\left(x_k^*\right)\left(x_k - x_{k-1}\right)$$

$$\text{Arc Length} = \sum_{k=1}^{n} \sqrt{1 + \left(f'\left(x_k^*\right)\right)^2}\,\Delta x$$

$$\text{Arc Length} = \lim_{\Delta x \to 0} \sum_{k=1}^{n} \sqrt{1 + \left(f'\left(x_k^*\right)\right)^2}\,\Delta x$$

$$\text{Arc Length} = \int_a^b \sqrt{1 + \left[f'(x)\right]^2}\,dx$$

Math 104 – Rimmer
9.1 Arc Length

Find the length of the curve.

$$y = \frac{2}{3}\left(1 + x^2\right)^{3/2} \quad 0 \le x \le 3$$

Find the length of the curve $y = 1 + \frac{2}{3}(x-1)^{3/2}$ for $1 \le x \le 4$.

Math 104 – Rimmer
9.1 Arc Length

Find the length of the curve.

$y = \frac{1}{3}\sqrt{x}(3-x) \quad 0 \le x \le 3 \qquad y = \sqrt{x} - \frac{1}{3}x^{3/2}$