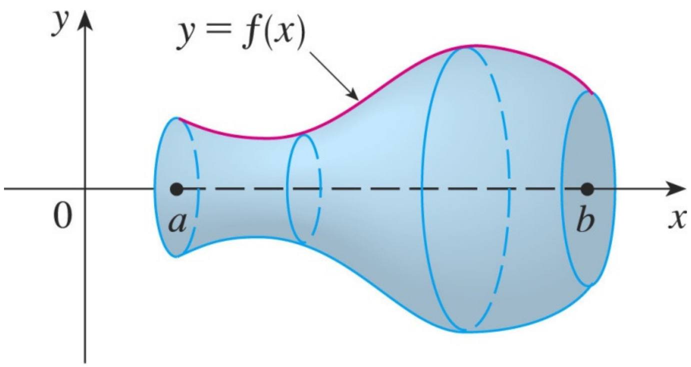
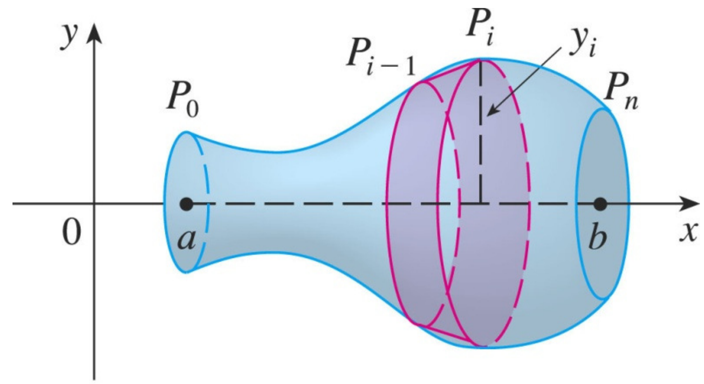


(a) Surface of revolution



(b) Approximating band

Area of the band $= 2\pi(\text{radius})(\text{length})$

For the radius, take an average.

For the length, take the distance from P_{i-1} to P_i.

$$= 2\pi\left(\frac{y_{i-1}+y_i}{2}\right)d\left(P_{i-1}P_i\right)$$

$$= 2\pi f\left(x_i^*\right)\sqrt{1+\left(f'\left(x_i^*\right)\right)^2}\,\Delta x$$

In 9.1 we saw $d\left(P_{i-1}P_i\right) = \sqrt{1+\left(f'\left(x_i^*\right)\right)^2}\,\Delta x$

For small Δx, $y_i = f\left(x_i\right) \approx f\left(x_i^*\right)$

and $y_{i-1} = f\left(x_{i-1}\right) \approx f\left(x_i^*\right)$

$$\text{Total surface area} = \sum_{i=1}^{n} 2\pi f\left(x_i^*\right)\sqrt{1+\left(f'\left(x_i^*\right)\right)^2}\,\Delta x$$

$$\underset{\text{(better approximation)}}{\text{Total surface area}} = \lim_{n\to\infty}\sum_{i=1}^{n} 2\pi f\left(x_i^*\right)\sqrt{1+\left(f'\left(x_i^*\right)\right)^2}\,\Delta x$$

the area of surface obtained by rotating the curve $y = f(x)$ about the $x-$axis for $a \le x \le b$ is

$$\text{Surface Area} = \int_a^b 2\pi f(x)\sqrt{1+\left(f'(x)\right)^2}\,dx$$

A function with a continuous derivative on $[a,b]$ $\Rightarrow$ the area of surface obtained by rotating the graph of a function about the $y-\text{axis}$ for $a \le x \le b$ is

$$SA = 2\pi \int_a^b x\,ds$$

function: $y = f(x)$

$$SA = 2\pi \int_a^b x\sqrt{1+\left[\frac{dy}{dx}\right]^2}\,dx$$

function: $x = g(y)$ $c \le y \le d$

$$SA = 2\pi \int_c^d g(y)\sqrt{1+\left[\frac{dx}{dy}\right]^2}\,dy$$

A function with a continuous derivative on $[a,b]$ $\Rightarrow$ the area of surface obtained by rotating the graph of a function about the $x-\text{axis}$ for $a \le x \le b$ is

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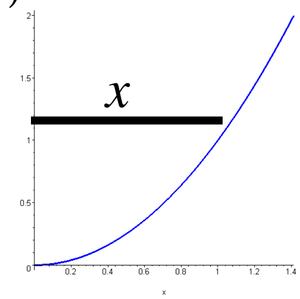
function: $x = g(y)$ $c \le y \le d$

$$SA = 2\pi \int_c^d y\sqrt{1+\left[\frac{dx}{dy}\right]^2}\,dy$$

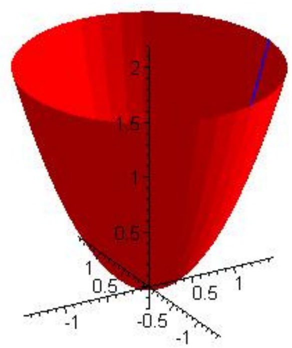
Find the area of the surface formed by revolving the graph of $f(x) = x^2$ on the interval $0 \le x \le \sqrt{2}$ about the *y*-axis.



$$SA = 2\pi \int_a^b x\sqrt{1+\left[\tfrac{dy}{dx}\right]^2}\,dx$$

$$f(x) = x^2 \Rightarrow \tfrac{dy}{dx} = 2x$$

$$\sqrt{1+\left[\tfrac{dy}{dx}\right]^2} = \sqrt{1+4x^2}$$



$$SA = 2\pi \int_0^{\sqrt{2}} x\sqrt{1+4x^2}\,dx$$

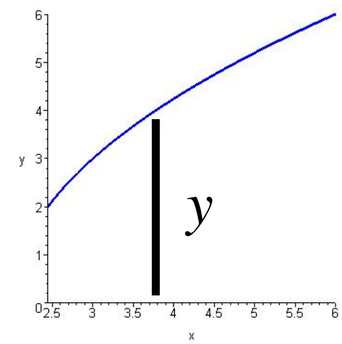
$$u = 1+4x^2$$

$$du = 8x\,dx \qquad \tfrac{1}{8}du = x\,dx$$

$$2\pi \tfrac{1}{8}\int u^{1/2}du = 2\pi \tfrac{1}{8}\cdot\tfrac{2}{3}u^{3/2} = \tfrac{\pi}{6}\left(1+4x^2\right)^{3/2}$$

$$SA = \tfrac{\pi}{6}\left[\left(1+4x^2\right)^{3/2}\right]_0^{\sqrt{2}} = \tfrac{\pi}{6}\left[9^{3/2}-1\right] = \frac{26\pi}{6} = \boxed{\frac{13\pi}{3}}$$

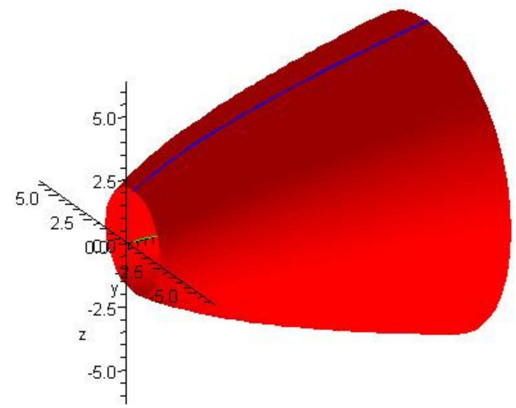
Find the area of the surface formed by revolving the graph of $x = \frac{1}{9}y^2 + 2$ on the interval $2 \le y \le 6$ about the x-axis.



$$SA = 2\pi \int_a^b y\sqrt{1 + \left[\tfrac{dx}{dy}\right]^2}\,dy$$

$$g(y) = \tfrac{1}{9}y^2 + 2 \Rightarrow \tfrac{dx}{dy} = \tfrac{2}{9}y$$

$$\sqrt{1 + \left[\tfrac{dx}{dy}\right]^2} = \sqrt{1 + \tfrac{4}{81}y^2} = \sqrt{\tfrac{81 + 4y^2}{81}} = \tfrac{1}{9}\sqrt{81 + 4y^2}$$



$$SA = \frac{2\pi}{9}\int_2^6 y\sqrt{81 + 4y^2}\,dy$$

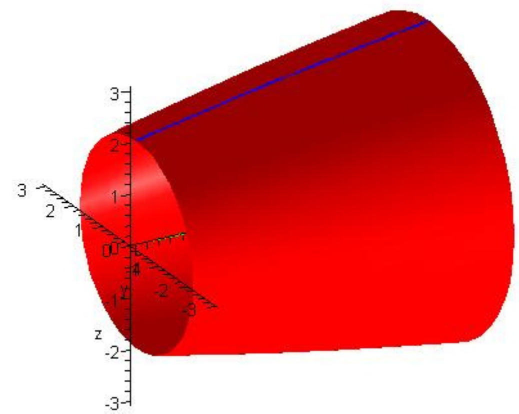
$$u = 81 + 4y^2$$

$$du = 8y\,dy \qquad \tfrac{1}{8}du = y\,dy$$

$$\tfrac{2\pi}{9}\tfrac{1}{8}\int u^{1/2}du = \tfrac{2\pi}{9}\tfrac{1}{8}\cdot\tfrac{2}{3}u^{3/2} = \tfrac{\pi}{54}\left(81 + 4y^2\right)^{3/2}$$

$$SA = \tfrac{\pi}{54}\left[\left(81 + 4y^2\right)^{3/2}\right]_2^6 = \boxed{\tfrac{\pi}{54}\left[225^{3/2} - 97^{3/2}\right]}$$

Find the area of the surface formed by revolving the graph of $f(x)=\sqrt{x}$ on the interval $4\le x\le 9$ about the x-axis.



axis: $x-\text{axis}$
function: $y=f(x)$

$$\Rightarrow SA=2\pi\int_a^b f(x)\sqrt{1+\left[\tfrac{dy}{dx}\right]^2}\,dx$$

$$f(x)=\sqrt{x}\Rightarrow \tfrac{dy}{dx}=\tfrac{1}{2\sqrt{x}}$$

$$\sqrt{1+\left[\tfrac{dy}{dx}\right]^2}=\sqrt{1+\tfrac{1}{4x}}=\sqrt{\frac{4x+1}{4x}}$$

$$SA=2\pi\int_4^9\sqrt{x}\sqrt{\frac{4x+1}{4x}}dx=2\pi\int_4^9\sqrt{\cancel{x}\cdot\frac{4x+1}{4\cancel{x}}}dx=\cancel{2}\pi\int_4^9\frac{1}{\cancel{2}}\sqrt{4x+1}dx=\pi\int_4^9\sqrt{4x+1}dx$$

$u=4x+1$

$du=4dx \quad \tfrac{1}{4}du=dx$

$\pi\cdot\tfrac{1}{4}\int u^{1/2}du=\pi\cdot\tfrac{1}{4}\cdot\tfrac{2}{3}u^{3/2}=\tfrac{\pi}{6}(4x+1)^{3/2}$

$$SA=\tfrac{\pi}{6}\left[(4x+1)^{3/2}\right]_4^9$$

$$=\boxed{\tfrac{\pi}{6}\left[37^{3/2}-17^{3/2}\right]}$$