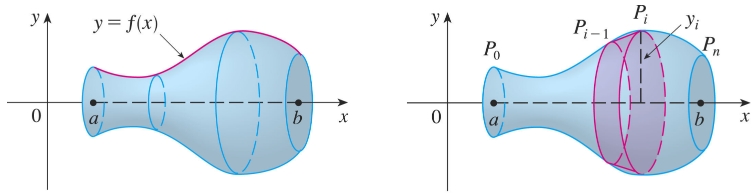


(a) Surface of revolution (b) Approximating band

$$\textbf{Area}\text{ of the band} = 2\pi(\text{radius})(\text{length})$$
$$= 2\pi\left(\frac{y_{i-1}+y_i}{2}\right)d\left(P_{i-1}P_i\right)$$
$$= 2\pi f'\left(x_i^*\right)\sqrt{1+\left(f'\left(x_i^*\right)\right)^2}\,\Delta x$$

For the radius, take an average.
For the length, take the distance from P_{i-1} to P_i.

In 9.1 we saw $d\left(P_{i-1}P_i\right)=\sqrt{1+\left(f'\left(x_i^*\right)\right)^2}\,\Delta x$

For small Δx, $y_i = f\left(x_i\right) \approx f\left(x_i^*\right)$

and $y_{i-1} = f\left(x_{i-1}\right) \approx f\left(x_i^*\right)$

$$\text{Total surface area} = \sum_{i=1}^{n} 2\pi f'\left(x_i^*\right)\sqrt{1+\left(f'\left(x_i^*\right)\right)^2}\,\Delta x$$

$$\underset{\text{(better approximation)}}{\text{Total surface area}} = \lim_{n\to\infty}\sum_{i=1}^{n} 2\pi f'\left(x_i^*\right)\sqrt{1+\left(f'\left(x_i^*\right)\right)^2}\,\Delta x$$

the area of surface obtained by rotating the curve $y = f\left(x\right)$ about the $x-$axis for $a \le x \le b$ is

$$\text{Surface Area} = \int_a^b 2\pi f\left(x\right)\sqrt{1+\left(f'\left(x\right)\right)^2}\,dx$$

Math 104 – Rimmer
9.2 Surface Area of Revolution

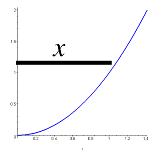
A function with a continuous derivative on $[a,b]$ $\Rightarrow$ the area of surface obtained by rotating the graph of a function about the $y-$axis for $a \le x \le b$ is

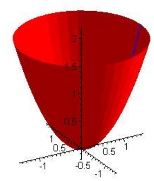
$$SA = 2\pi\int_a^b x\,ds$$

A function with a continuous derivative on $[a,b]$ $\Rightarrow$ the area of surface obtained by rotating the graph of a function about the $x-$axis for $a \le x \le b$ is

$$SA = 2\pi\int_a^b y\,ds$$

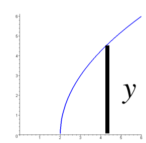
Find the area of the surface formed by revolving the graph of $f(x) = x^2$ on the interval $\left[0, \sqrt{2}\right]$ about the y-axis.

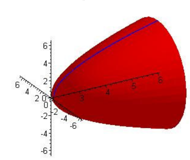




Math 104 – Rimmer
9.2 Surface Area of Revolution

Find the area of the surface formed by revolving the graph of $x = \frac{1}{9}y^2 + 2$ on the interval $[2, 6]$ about the x-axis.





Find the area of the surface formed by revolving the graph of $f(x)=\sqrt{x}$ on the interval $[4,9]$ about the x-axis.