

### 9.5 Probability Density Function

A \_\_\_\_\_, usually written  $X$ , is a variable whose possible values are numerical outcomes of a random phenomenon.

A \_\_\_\_\_ is one which takes an infinite number of possible values (usually measurements)

A \_\_\_\_\_ is a function  $f$  defined for all real  $x$  and having the following properties:

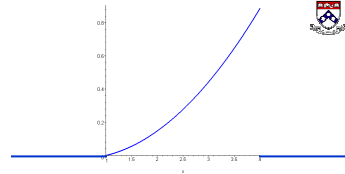
- 1.
- 2.

» Every continuous random variable,  $X$ , \_\_\_\_\_.

» Used to determine \_\_\_\_\_ that a continuous random variable lies between two values

$$P(a \leq X \leq b) =$$

$$f(x) = \begin{cases} \frac{2}{27}x(x-1) & 1 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$



a) Verify that  $f$  is a probability function.

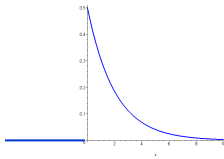
b) Find  $P(2 \leq x \leq 3)$ .

$$f(x) = \begin{cases} kx^2 & 2 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

Find  $k$  such that  $f(x)$  is a probability density function.

### Exponential Density Function

$$f(x) = \begin{cases} 0 & x < 0 \\ ke^{-kx} & x \geq 0 \end{cases}, k > 0$$



### Examples of applications:

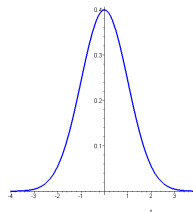
- » life span of electronic components
- » duration of telephone calls
- » waiting time in a doctor's office
- » time b/w successive flight arrivals and departures in an airport

### Normal Density Function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$$

$$\mu =$$

$$\sigma =$$



**Mean** (average value) or expected value of a probability density function is a measure of the center of a pdf.

$$\mu =$$

$$f(x) = \begin{cases} \frac{2}{27}x(x-1) & 1 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

Find the mean.

$$f(x) = \begin{cases} 0 & x < 0 \\ ke^{-kx} & x \geq 0, k > 0 \end{cases}$$

Find the mean.

**Median** ( $m$ ) of a probability density function is a number such that  $\frac{1}{2}$  the area under the graph of  $f$  lies to the right of it.

$$f(x) = \begin{cases} \frac{1}{39}x^2 & 2 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

Find the median.

