12.2

Test for Divergence :

If $\lim_{n\to\infty} a_n \neq 0$ or $\lim_{n\to\infty} a_n$ does not exist, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

geometric series $\sum_{n=1}^{\infty} ar^{n-1}$ or $\sum_{n=1}^{\infty} ar^{n}$ is convergent for $|r| < 1$ and divergent if $|r| \geq 1$.

12.3 The Integral Test

If $f(x)$ is: $a)$ continuous, on the interval $\underset{\text{constant } k>0}{[k,\infty)}$
$b)$ positive,
$c)$ and decreasing

,then the series $\sum_{n=k}^{\infty} a_n$ $\left(\text{with } a_n = f(n)\right)$

$i)$ is convergent when $\int_k^{\infty} f(x)\,dx$ is convergent.

$ii)$ is divergent when $\int_k^{\infty} f(x)\,dx$ is divergent.

p – series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent for $p > 1$ and divergent if $p \leq 1$.

12.4

The Comparison Test:

Given the series $\sum_{n=1}^{\infty} a_n, \ (a_n \geq 0)$

(i) if the terms a_n are **smaller** than the terms b_n of a known **convergent** series $\sum_{n=1}^{\infty} b_n \ (b_n \geq 0)$, then our series $\sum_{n=1}^{\infty} a_n$ is also **convergent**.

(ii) if the terms a_n are **larger** than the terms b_n of a known **divergent** series $\sum_{n=1}^{\infty} b_n \,(b_n \geq 0)$, then our series $\sum_{n=1}^{\infty} a_n$ is also **divergent**.

12.4

The Limit Comparison Test:

Given the series $\sum_{n=1}^{\infty} a_n, \ (a_n > 0)$ and a known convergent or divergent series $\sum_{n=1}^{\infty} b_n, \ (b_n > 0)$

If the $\lim_{n\to\infty} \frac{a_n}{b_n} = c$ where c is a finite positive number, then the series will behave alike, i.e. either both converge or both diverge.

12.5 The Alternating Series Test

If the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} b_n \ (\text{where } b_n > 0)$ satisfies:

$i)$ $\lim_{n\to\infty} b_n = 0$

$ii)$ $\{b_n\}$ is a decreasing sequence, and

,then the series is **convergent**.

12.6 The Ratio Test

Let $\{a_n\}$ be a sequence and assume that the following limit exists: $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L$

$i)$ If $L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.

$ii)$ If $L > 1$ or if the limit is infinite, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

$iii)$ If $L = 1$, the Ratio Test is inconclusive.

(the series could be absolutely convergent, conditionally convergent, or divergent)

12.6 The Root Test

Let $\{a_n\}$ be a sequence and assume that the following limit exists: $\lim_{n\to\infty}\sqrt[n]{|a_n|} = L$

$i)$ If $L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.

$ii)$ If $L > 1$ or if the limit is infinite, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

$iii)$ If $L = 1$, the Root Test is inconclusive.

(the series could be absolutely convergent, conditionally convergent, or divergent)