10.1 Sequences

A ___________________ is an ordered list of numbers.

A sequence can be _____ or __________.

In this class we will deal with _______ sequences

Note: the sequence doesn't have to start at *n* = 1

Notation:

$$\{a_1, a_2, a_3, \ldots, a_n, a_{n+1}, \ldots\}$$

$$\{a_n\} \quad \text{or} \quad \{a_n\}_{n=1}^{\infty}$$

a formula for the n^{th} term

$$\left\{\frac{1}{4}, \frac{2}{9}, \frac{3}{16}, \frac{4}{25}, \frac{5}{36}, \ldots\right\}$$

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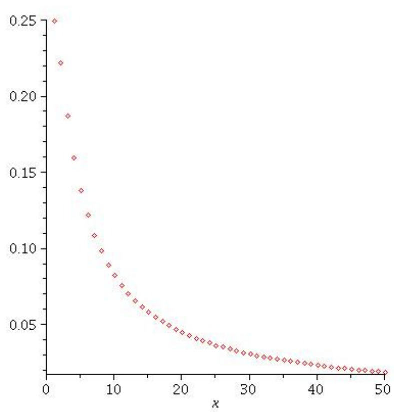
$$a_n = \frac{n}{(n+1)^2}$$

input :

output :

$$\left(1, \frac{1}{4}\right), \left(2, \frac{2}{9}\right), \left(3, \frac{3}{16}\right), \left(4, \frac{4}{25}\right), \left(5, \frac{5}{36}\right), \ldots$$

These isolated points make up the graph of the sequence.



It seems as though the terms of the sequence are approaching ____ as $n \to \infty$

$$\lim_{n\to\infty} \frac{n}{(n+1)^2}$$

$$\lim_{n\to\infty} \frac{p(n)}{q(n)} = 0 \quad (p \text{ and } q \text{ polynomials})$$

when ______________________.

In general, if the terms of the sequence are approaching L as $n \to \infty$, then $\lim_{n\to\infty} a_n = L$

When this limit exists and is finite, we say the sequence is __________.

When the $\lim_{n\to\infty} a_n$ does not exist or is infinite, the sequence is called ________.

$$\left\{\cos\left(\frac{n\pi}{2}\right)\right\}$$

$\Rightarrow \left\{\cos\left(\frac{n\pi}{2}\right)\right\}$ is ______ since the $\lim_{n\to\infty}\cos\left(\frac{n\pi}{2}\right)$ __________.

$$\left\{\frac{n^2}{n+2}\right\} \qquad \lim_{n\to\infty}\frac{n^2}{n+2} = \infty$$

$\lim_{n\to\infty}\frac{p(n)}{q(n)} = \infty$ (p and q polynomials)

when __________________.

$\Rightarrow \left\{\frac{n^2}{n+2}\right\}$ is ___________.

So basically, finding the limit of a sequence boils down to being able to find limits at infinity.

Tools:

Section 2.2 Limit Laws **Section 4.4 Limits at Infinity**

Section 7.8 Indeterminate forms and L'Hopitals Rule

Thoerems:

1. Squeeze Theorem:

$$\left.\begin{array}{l} a_n \le c_n \le b_n \ \text{ for all } n > N \\ \text{and} \\ \lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = L \end{array}\right\} \Rightarrow \lim_{n\to\infty} c_n = L$$

2. $\lim_{n\to\infty}|a_n| = 0 \Rightarrow \lim_{n\to\infty} a_n = 0$

3. The sequence $\{r^n\}$ is $\begin{cases} \text{convergent} & \text{if } -1 < r \le 1 \\ \text{divergent} & \text{for all other values of } r \end{cases}$

$$\lim_{n\to\infty} r^n = \begin{cases} 0 & \text{if } -1 < r < 1 \\ 1 & \text{if } r = 1 \end{cases}$$

4. $$\left.\begin{array}{l} \lim_{n\to\infty} a_n = L \\ \text{and} \\ f \text{ is contin. at } L \end{array}\right\} \Rightarrow \lim_{n\to\infty} f(a_n) = f\left(\lim_{n\to\infty} a_n\right) = f(L)$$

bring the limit inside

5. Every **bounded** and **increasing** sequence and every **bounded** and **decreasing** sequence is **convergent**.

Determine whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = \frac{3+5n^2}{n+n^2}$$

$$a_n = \left(\frac{1}{\pi}\right)^n$$

Determine whether the sequence converges or diverges.
If it converges, find the limit.

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$$a_n = \sqrt{\frac{n+1}{9n+1}}$$

$$a_n = \frac{(-1)^{n-1} n}{n^2+1}$$

Determine whether the sequence converges or diverges.

If it converges, find the limit.

$$a_n = \left(\frac{n+1}{n}\right)^n$$

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Determine whether the sequence converges or diverges.
If it converges, find the limit.

$$a_n = \frac{(-1)^n \sin\left(n^2\right)}{n}$$