10.4 Comparison Tests

The Direct Comparison Test:

Given the series $\sum_{n=1}^{\infty} a_n, \ (a_n \geq 0)$

(i) if the terms a_n are ________ than the terms b_n of a known __________

series $\sum_{n=1}^{\infty} b_n \ (b_n \geq 0)$, then our series $\sum_{n=1}^{\infty} a_n$ is also ___________.

(ii) if the terms a_n are _______ than the terms b_n of a known ___________

series $\sum_{n=1}^{\infty} b_n \ (b_n \geq 0)$, then our series $\sum_{n=1}^{\infty} a_n$ is also __________.

For the series $\sum_{n=1}^{\infty} b_n$, it must be known whether it converges or diverges, so it is usually chosen to be a ______ or a _____________.

search for the ____________ in both the numerator and the denominator of a_n,
choose your b_n to be the ratio of these ___________

Consider the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1} \cdot 4^n}$

Consider the series $\sum_{n=9}^{\infty} \frac{\sqrt{n}}{n-8}$

The inequality $a_n \le b_n$ or $b_n \le a_n$ doesn't need to be satisfied for all values of n.
If it doesn't hold for the first few terms but it holds for all $n > N$ for some N, then the comparison test will still work.

Consider the series $\sum_{n=1}^{\infty} \frac{\ln n}{n}$ Choose $b_n = \frac{1}{n}$ $\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n}$ is the harmonic series, so it is divergent

Since $\sum_{n=1}^{\infty} b_n$ is divergent, the inequality should be $\frac{1}{n} \le \frac{\ln n}{n}$

$\Rightarrow n \le n \ln n \;\; \Rightarrow \frac{n}{n} \le \frac{n \ln n}{n} \;\; \Rightarrow 1 \le \ln n \;\; \Rightarrow e^1 \le e^{\ln n} \;\; \Rightarrow n > e$

The inequality doesn't hold for $n = 1$ or $n = 2$ but it holds for all $n \ge 3$

The convergence or divergence of the series does not depend on the first two terms. These terms can be subtracted off and we can look at both series starting at $n = 3$.

$\sum_{n=1}^{\infty} \frac{1}{n}$ diverges $\Rightarrow \sum_{n=1}^{\infty} \frac{\ln n}{n}$ also diverges by the Comparison Test

Consider the series $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3+1}}$

The Limit Comparison Test:

Given the series $\sum_{n=1}^{\infty} a_n,\ (a_n > 0)$ and a known

convergent or divergent series $\sum_{n=1}^{\infty} b_n,\ (b_n > 0)$

If the $\lim_{n\to\infty} \frac{\quad}{\quad} = c$ where c is a finite positive number, then

the series will

If the $\lim_{n\to\infty} \frac{\quad}{\quad} = 0$ and

If the $\lim_{n\to\infty} \frac{\quad}{\quad} = \infty$ and

Back to the series $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3+1}}$.

Consider the series $\sum_{n=1}^{\infty} \frac{1+3^n}{4+2^n}$

Consider the series $\sum_{n=1}^{\infty} \frac{3n+4}{(2n+1)^3}$