10.6 Alternating Series Test

An **alternating series** is of the form $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (-1)^{n-1} b_n$ or $\sum_{n=1}^{\infty} (-1)^n b_n$, (where $b_n > 0$)

(it has successive terms of opposite signs) $\qquad b_n = |a_n|$

Example: $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} =$

Example: $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n+5} =$

Forms for the term that makes the series alternate in sign:

10.6 The Alternating Series Test

If the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$ (where $b_n > 0$) satisfies:

$i)$ $\lim_{n\to\infty} b_n = 0$

$ii)$ $\{b_n\}$ is a decreasing sequence, and

,then the series is **convergent**.

<u>Note:</u>

$a)$ This test is for convergence only. It says nothing about divergence.

$b)$ Like the function in the Integral Test, the sequence $\{b_n\}$ needs to be decreasing "eventually" i.e., for all $n > N$ for some N

Example 1:

$$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}$$

Example 2:

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}n^2}{n^2+5}$$

Example 3:

$$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\ln n}{n}$$

10.6 Absolute Convergence

An infinite series

$\sum_{n=1}^{\infty} a_n$ is called ____________________ if the positive series $\sum_{n=1}^{\infty} |a_n|$ converges.

___________________ implies _______________.

(If the series of absolute value converges, then the original series also converges)

If the series of absolute value ________, it is still possible for the original series to converge.

Use the ___________________ on the original series.
If the Alternating Series Test gives convergence, then this is a special type of convergence.

An infinite series

$\sum_{n=1}^{\infty} a_n$ is called _____________________ if it converges but $\sum_{n=1}^{\infty} |a_n|$ diverges.

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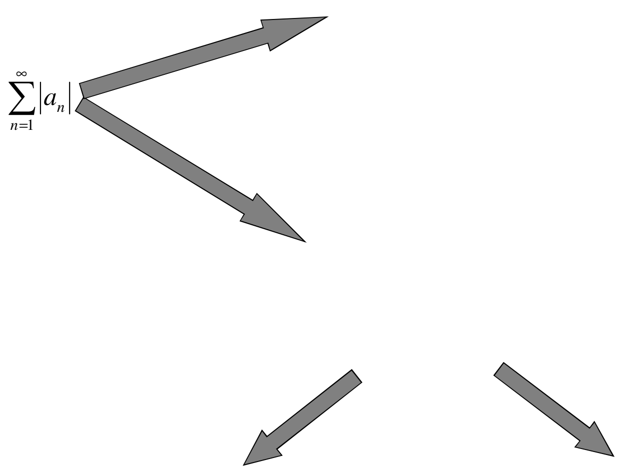
10.6 Alternating Series and
Absolute Convergence



A major difference between absolutely convergent and conditionally convergent comes in the ______________________.

If $\sum_{n=1}^{\infty} a_n$ is ____________________ with sum s,
then any rearrangement of the sum $\sum_{n=1}^{\infty} a_n$ will ________________.

If $\sum_{n=1}^{\infty} a_n$ is __________________ and r is any real number,
then there is a rearrangement of the sum $\sum_{n=1}^{\infty} a_n$ that ____________.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots = \ln 2 \quad (\text{We will show this later})$$

$$\frac{1}{2}\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots\right) = \frac{1}{2}\ln 2$$

$$\frac{1}{2} - \frac{1}{4} + \frac{1}{6} + \cdots = \frac{1}{2}\ln 2$$

$$0 + \frac{1}{2} + 0 - \frac{1}{4} + 0 + \frac{1}{6} + \cdots = \frac{1}{2}\ln 2$$

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8} + \cdots = \ln 2$$

$$+ \quad 0 + \frac{1}{2} + 0 - \frac{1}{4} + 0 + \frac{1}{6} + 0 + -\frac{1}{8} + \cdots = \frac{1}{2}\ln 2$$

$$1 + \quad \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \quad \frac{1}{7} - \frac{1}{4} + \cdots = \frac{3}{2}\ln 2$$

different sums

same terms

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10.6 Alternating Series and Absolute Convergence

Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$i)\ \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\sqrt{n^3}}$

$ii)\ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$

$iii)\ \sum_{n=1}^{\infty} \frac{(-4)^{n+1}}{3^n}$