10.7 Power Series

A power series is a series of the form

$$\sum_{n=0}^{\infty} c_n x^n =$$

where:

$a)$

$b)$

For each fixed x, the series above is a series of constants that we can test for convergence or divergence.

A power series may converge for some values of x and diverge for other values of x.

The sum of the series is a function

whose _________ is the set of all x for which the series converges.

$f(x)$ is reminiscent of a __________ but it has infinitely many terms

If all c_n's $= 1$, we have

$$f(x) = 1 + x + x^2 + \ldots + x^n + \ldots = \sum_{n=0}^{\infty} x^n$$

This is the ________________ with _____.

The power series will converge for ____ and diverge for all other x.

In general, a series of the form

is called a power series ______ or a power series about a

We use the ________________ to find for what values of x the series converges.

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L \text{ \_\_\_\_\_\_\_\_\_\_}$$

solve for $|x-a|$ to get $|x-a|<R$

$$\Rightarrow -R < x-a < R$$

$$\Rightarrow a-R < x < a+R$$

R is called the ________ ____________ (R.O.C.).

This is called the _______ ______________ (I.O.C.).

Plug in the endpoints to check for convergence or divergence at the endpoints.

Find the radius of convergence and the interval of convergence.

$$\sum_{n=1}^{\infty}\frac{(-1)^n n^2 x^n}{2^n}$$

$x=$ ___ $x=$ ___

Radius of convergence: ______
Interval of convergence: _______

Find the radius of convergence and the interval of convergence.

$$\sum_{n=1}^{\infty} \frac{3^n (x+4)^n}{\sqrt{n}}$$

$x =$ ___ $x =$ ___

R.O.C.: _______

I.O.C. : _______

Find the radius of convergence and the interval of convergence.

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$$\sum_{n=1}^{\infty} \frac{(4x+1)^n}{n^2}$$

Check endpoints:

$x =$ ___ $x =$ ___

R.O.C.: ______

I.O.C. : ______

Sometimes the Root Test can be used just as the Ratio Test.

When a_n can be written as $(b_n)^n$, then the Root Test should be used.

$$\sum_{n=1}^{\infty} \frac{3^n (x-5)^n}{n^n}$$

R.O.C. = _____
I.O.C. = _____

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 0 \Rightarrow$$

$$\sum_{n=1}^{\infty} \frac{n!(x-7)^n}{2^n}$$

R.O.C. = ____
I.O.C. = ____

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \infty \Rightarrow$$

Find the radius of convergence.

$$\sum_{n=1}^{\infty} \frac{(-1)^n (n!)^2 x^{2n}}{(2n)!}$$

Radius of convergence: ____

Functions as Power Series

The very first function we have seen represented as a power series is the geometric series with $a = 1$ and $r = x$

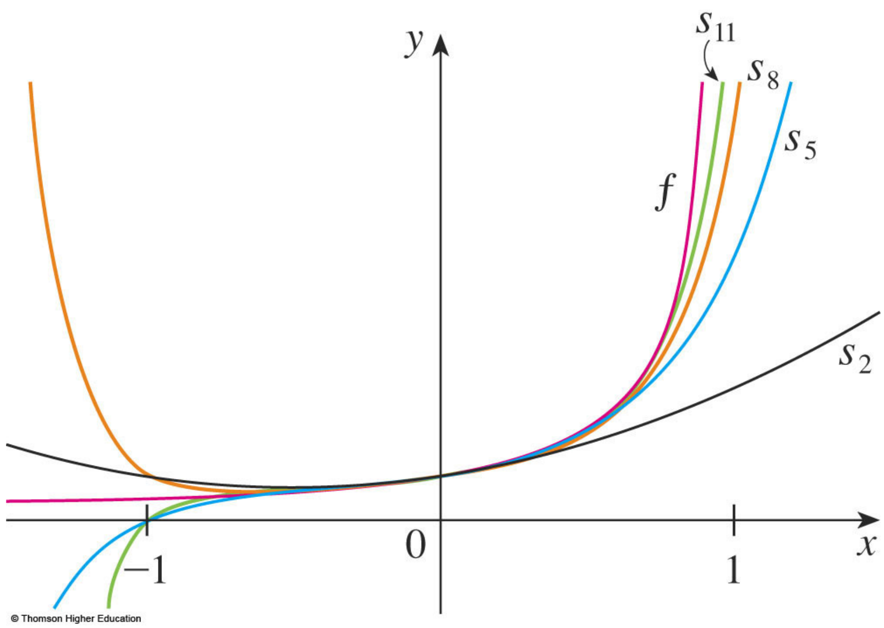
$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots \quad , |x| < 1$$

We can find the power series representation of other functions by algebraically manipulating them to to be some multiple of this series.

$$\frac{1}{1+x} = \frac{1}{1-(-x)}$$

The interval of convergence remains unchanged since this is still a type of geometric series.

$$\frac{1}{1+x} =$$



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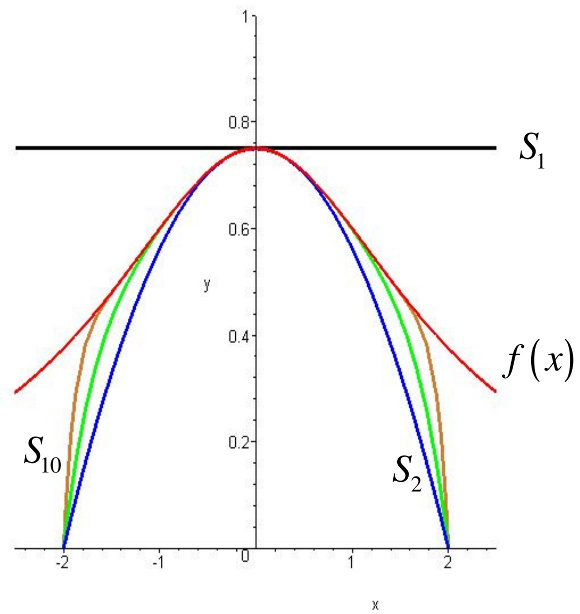
Represent the function as a power series and determine the interval of convergence.

$$\frac{3}{4+x^2}$$

, so the interval of convergence is

$$\frac{3}{4+x^2} =$$



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$$f(x)=\sum_{n=0}^{\infty}c_n(x-a)^n=c_0+c_1(x-a)+c_2(x-a)^2+c_3(x-a)^3+\cdots$$

If the power series representation of $f(x)$ has a radius of convergence $R>0$,

we can obtain a power series representation for $f'(x)$ by

term - by - term ____________:

$$f(x)=c_0+c_1(x-a)+c_2(x-a)^2+c_3(x-a)^3+\cdots$$

$$f'(x)=c_1+2c_2(x-a)+3c_3(x-a)^2+\cdots$$

$$f'(x)=\frac{d}{dx}\left(\sum_{n=0}^{\infty}c_n(x-a)^n\right)=\sum_{n=0}^{\infty}\frac{d}{dx}\left[c_n(x-a)^n\right]=\sum_{n=1}^{\infty}nc_n(x-a)^{n-1}$$

with the same radius of convergence R

starts at $n=1$

we can obtain a power series representation for $\int f(x)dx$ by

term - by - term ___________:

$$\int f(x)\,dx=C+c_0x+c_1\frac{(x-a)^2}{2}+c_2\frac{(x-a)^3}{3}+c_3\frac{(x-a)^4}{4}+\cdots$$

$$\int\left(\sum_{n=0}^{\infty}c_n(x-a)^n\right)dx=\sum_{n=0}^{\infty}\int\left[c_n(x-a)^n\right]dx\;=C+\sum_{n=0}^{\infty}\frac{c_n(x-a)^{n+1}}{n+1}$$

with the same radius of convergence R

C is a constant of integration

Represent the function as a power series and determine the radius of convergence.

$$f(x) = \frac{x^3}{(1-x)^2}$$

$$g(x) = \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad \text{with } R = 1$$

Represent the function as a power series and determine the radius of convergence.

$$f(x) = \arctan x$$

$$\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n}, \quad \text{with } R = 1$$

$$= 1 - x^2 + x^4 - x^6 + \ldots$$

$$\arctan x = C +$$

$$\arctan x =$$

Represent the function as a power series and determine the radius of convergence.

$f(x) = \ln(1-x)$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad \text{with } R = 1$$

$-\ln(1-x) =$

$\ln(1-x) =$

$$\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \quad \text{,with } R = 1$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$$

$\arctan 1 =$

$$\ln(1-x) = -\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1} \quad \text{,with } R = 1$$

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

$\ln\left(1-\frac{1}{2}\right) =$

$\ln\left(\frac{1}{2}\right) =$

$\ln 1 - \ln 2 =$

Algebraically manipulate $\dfrac{1}{(1-x)^2}$ $\left(\text{the same way we manipulated } \dfrac{1}{1-x}\right)$

$$\frac{1}{(1-x)^2} = \sum_{n=1}^{\infty} n x^{n-1} \quad \text{with } R = 1$$

Represent $\dfrac{1}{(4-3x)^2}$ as a power series and determine the radius of convergence.

$$\frac{1}{(4-3x)^2}$$

Algebraically manipulate $\ln(1-x)$ $\left(\text{the same way we manipulated } \dfrac{1}{1-x}\right)$

$$\ln(1-x) = -\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1} \text{ ,with } R = 1$$

Represent $\ln(3+2x)$ as a power series and determine the radius of convergence.

$$\ln(3+2x)$$