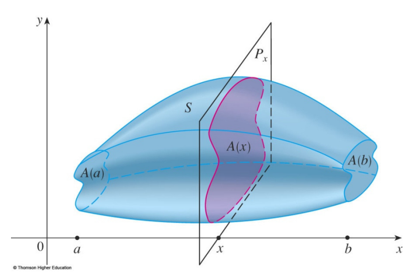
Goal: To find the volume of a solid using second semester calculus

- Volume by Cross-Sections
- Volume by Disk
- Volume by Washer

→ **Volume by Slicing**

- Volume by Shells

6.1 Volumes by Slicing



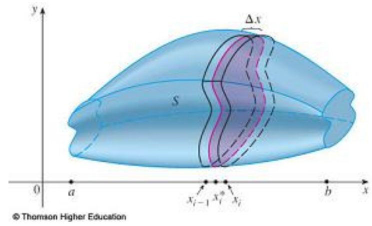
Goal: To find the volume of a solid

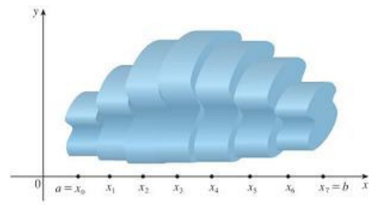
Method: "Cutting" the solid into many "pieces", find the volume of the pieces and add to find the total volume.

- The "pieces" are treated are cylinders.
- The base of each cylinder is called a **cross-section**.

6.1 Volumes

• The volume of each cylinder is found by taking the area of the cross-section, $A(x_i^*)$, and multiplying by the height, Δx.





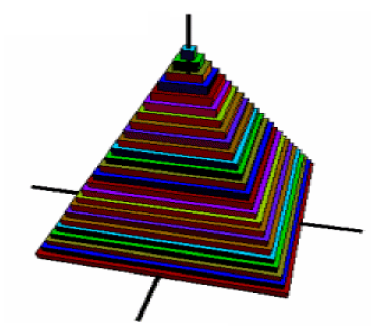
• The volume of the solid can be approximated by the sum of all cylinders.

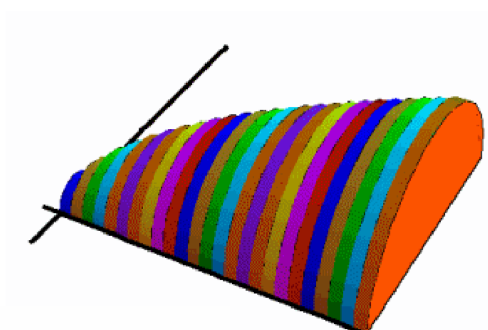
$$V \approx \sum_{i=1}^{n} A\left(x_i^*\right)\Delta x$$

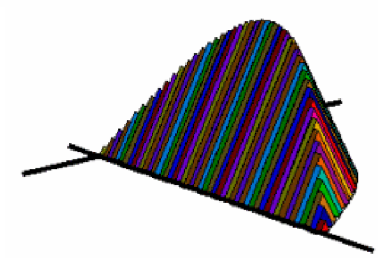
• Taking the limit as the number of cylinders goes to infinity gives the exact volume of the solid.

$$V = \lim_{n\to\infty}\sum_{i=1}^{n} A\left(x_i^*\right)\Delta x \qquad \Rightarrow V = \int_a^b A(x)\,dx$$

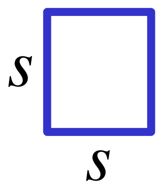
http://www.mathdemos.org/mathdemos/sectionmethod/sectiongallery.html





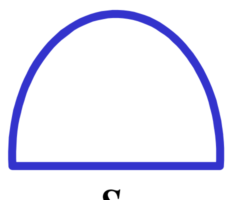


Area of a square with side length s :



$$Area = s^2$$

Area of a semicircle with diameter length s :



$$Area = \frac{1}{2}\pi\left(\frac{s}{2}\right)^2 = \frac{\pi}{2}\left(\frac{s^2}{4}\right)$$

$$r = \frac{s}{2}$$

$$Area = \frac{\pi}{8}s^2$$

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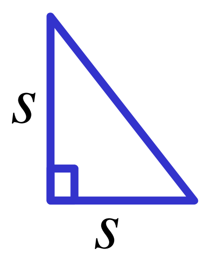
Area of an isosceles right triangle with leg length s :

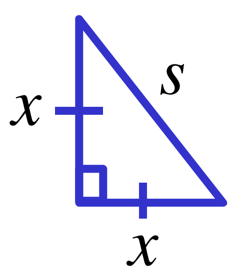


$$base = s$$

$$height = s$$

$$Area = \frac{1}{2}s^2$$

Area of an isosceles right triangle with hypotenuse length s :



$$x^2 + x^2 = s^2$$

$$2x^2 = s^2$$

$$x^2 = \frac{s^2}{2}$$

$$x = \frac{s}{\sqrt{2}}$$

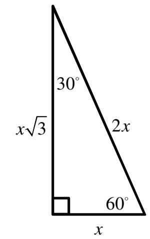
$$base = \tfrac{s}{\sqrt{2}}$$

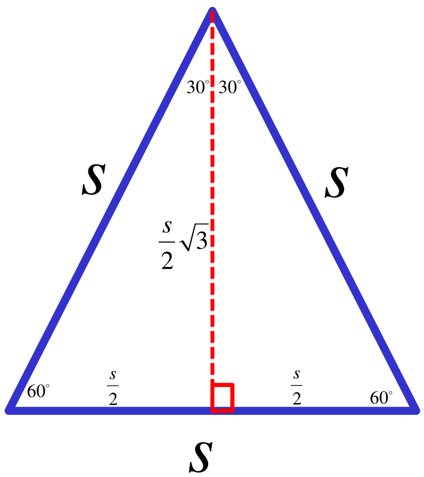
$$height = \tfrac{s}{\sqrt{2}}$$

$$Area = \frac{1}{2}\left(\tfrac{s}{\sqrt{2}}\right)\left(\tfrac{s}{\sqrt{2}}\right) = \frac{s \cdot s}{2 \cdot 2}$$

$$Area = \frac{1}{4}s^2$$

Area of a equilateral triangle with side length s :





$base = s$

$height = \frac{s}{2}\sqrt{3}$

$$Area = \frac{1}{2}(s)\left(\frac{s}{2}\sqrt{3}\right) = \frac{s \cdot s \cdot \sqrt{3}}{4}$$

$$Area = \frac{\sqrt{3}}{4}s^2$$

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Some Area Formulas

Area of a square with side length s : $Area = s^2$

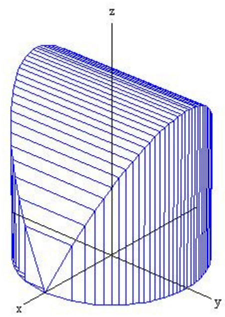
Area of a semicircle with diameter length s : $Area = \frac{\pi}{8}s^2$

Area of an isosceles right triangle with leg length s : $Area = \frac{1}{2}s^2$

Area of an isosceles right triangle with hypotenuse length s : $Area = \frac{1}{4}s^2$

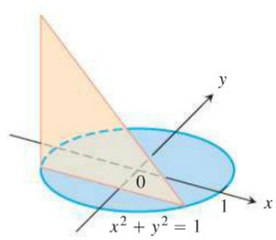
Area of a equilateral triangle with side length s : $Area = \frac{\sqrt{3}}{4}s^2$

A solid has a circular base of radius 4. If every plane cross section perpendicular to the x-axis is a square, then find the volume of the solid.



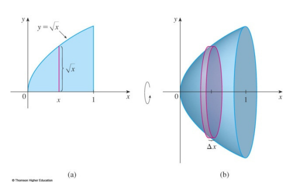
Section 6.1

10. The base of the solid is the disk $x^2 + y^2 \leq 1$. The cross-sections by planes perpendicular to the y-axis between $y = -1$ and $y = 1$ are isosceles right triangles with one leg in the disk.

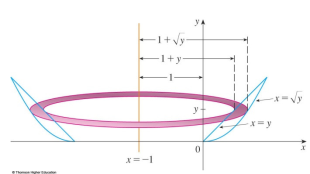


When you revolve a plane region about an axis, the cross-sections are circular and the solid generated is called a **solid of revolution**.

If there is **no gap** between the axis of rotation and the region, then the method used is called the **disk method**.



If there is **a gap** between the axis of rotation and the region, then the method used is called the **washer method**.

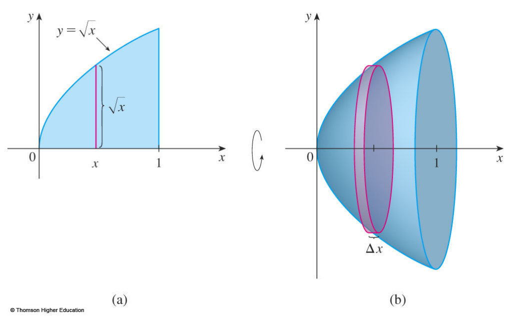


Disk Method with **horizontal axis of rotation** (not necessarily the x-axis)

Cross-sections are circular: $A(x)=\pi\left[r(x)\right]^2$ (radius as a function of x)

$$Volume=\int_a^b A(x)\,dx=\pi\int_a^b\left[r(x)\right]^2dx$$

(radius as a function of x)



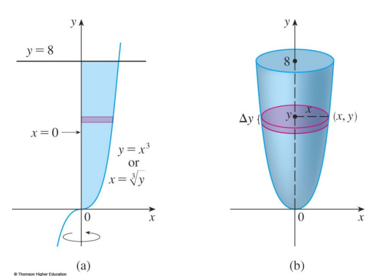
(a) (b)

Disk Method with **vertical axis of rotation** (not necessarily the y-axis)

Cross-sections are circular: $A(y)=\pi\left[r(y)\right]^2$ (radius as a function of y)

$$Volume=\int_a^b A(y)\,dy=\pi\int_a^b\left[r(y)\right]^2dy$$

(radius as a function of y)



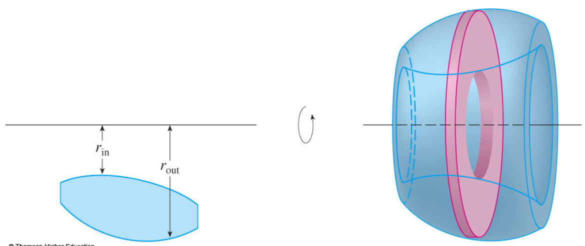
(a) (b)

Washer Method with **horizontal axis of rotation** (not necessarily the x-axis)

Draw a radius from the axis of rotation to the outer curve and call this **outer radius**

Draw a radius from the axis of rotation to the inner curve and call this **inner radius**

$$Volume = \int_a^b A(x)\,dx = \pi \int_a^b \left(\underbrace{\left[r_{out}(x)\right]^2}_{\text{outer radius as a function of } x} - \underbrace{\left[r_{in}(x)\right]^2}_{\text{inner radius as a function of } x} \right) dx$$



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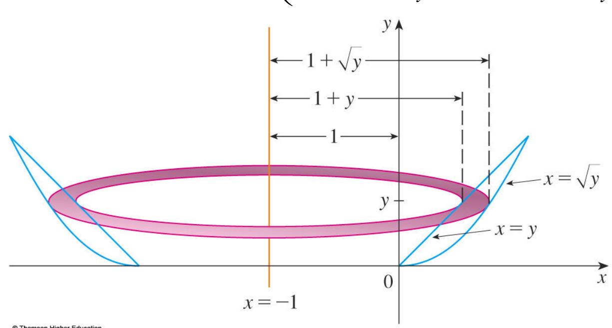
Washer Method with **vertical axis of rotation** (not necessarily the y-axis)

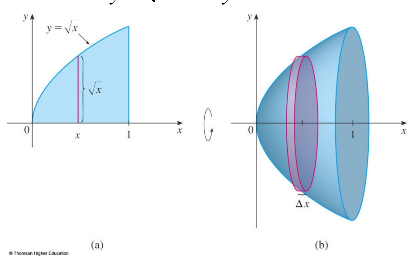
Draw a radius from the axis of rotation to the outer curve and call this **outer radius**

Draw a radius from the axis of rotation to the inner curve and call this **inner radius**

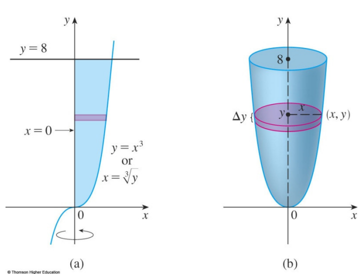
$$Volume = \int_a^b A(y)\,dy = \pi \int_a^b \left(\underbrace{\left[r_{out}(y)\right]^2}_{\text{outer radius as a function of } y} - \underbrace{\left[r_{in}(y)\right]^2}_{\text{inner radius as a function of } y} \right) dy$$



Calculate the volume of the solid generated by rotating the region between the curves $y = \sqrt{x}$ and $y = 0$ about the x-axis.



Calculate the volume of the solid generated by rotating the region between the curves $y = x^3$, $y = 8$, and $x = 0$ about the y-axis.



Calculate the volume of the solid generated by rotating the region between the curves $y = 4 - x^2$ and $y = 0$ **about the** x**-axis.**

Calculate the volume of the solid generated by rotating the region between the curves $y = \frac{1}{x-2}$ and $x = 2,\ y = \frac{1}{2},$ and $y = 4$ **about the line** $x = -4$

Calculate the volume of the solid generated by rotating the region between the curves $y = 4 - x^2$ and $y = 0$ **about the line** $y = -2$

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Calculate the volume of the solid generated by rotating the region between the curves $y = \frac{x}{2}$ and $y = \sqrt{x}$ **about the *y*-axis.**