

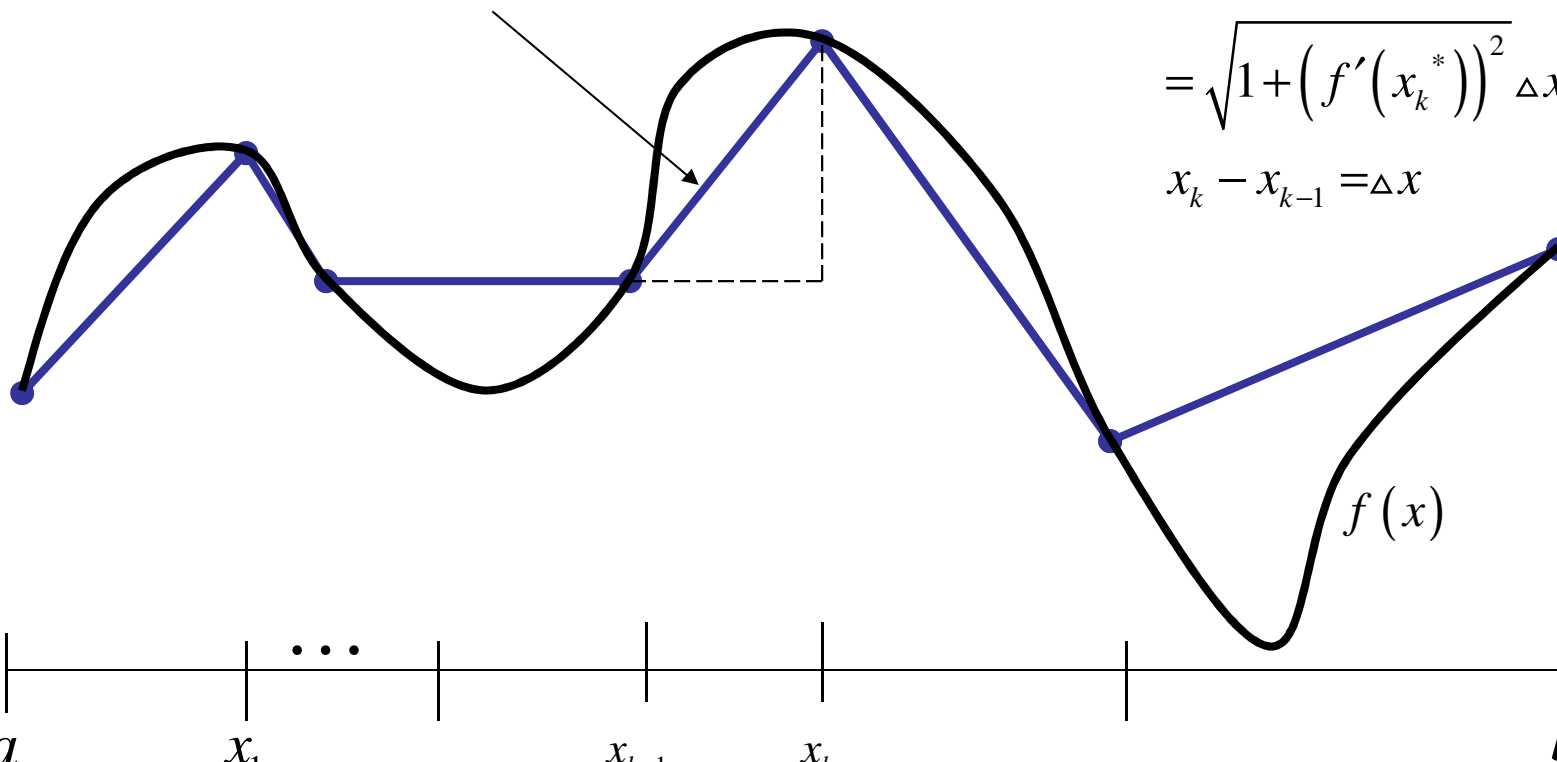
# 6.3 Arc Length



$$\sqrt{(x_k - x_{k-1})^2 + (f(x_k) - f(x_{k-1}))^2} = \sqrt{(x_k - x_{k-1})^2 + (f'(x_k^*)(x_k - x_{k-1}))^2}$$

$$= \sqrt{1 + (f'(x_k^*))^2} \Delta x$$

$$x_k - x_{k-1} = \Delta x$$



by the Mean Value Theorem:  $\exists x_k^* \in (x_{k-1}, x_k)$  such that

$$\frac{f(x_k) - f(x_{k-1})}{x_k - x_{k-1}} = f'(x_k^*)$$

$$\Rightarrow f(x_k) - f(x_{k-1}) = f'(x_k^*)(x_k - x_{k-1})$$

$$\text{Arc Length} = \sum_{k=1}^n \sqrt{1 + (f'(x_k^*))^2} \Delta x$$

$$\text{Arc Length} = \lim_{\Delta x \rightarrow 0} \sum_{k=1}^n \sqrt{1 + (f'(x_k^*))^2} \Delta x$$

$$\text{Arc Length} = \int_a^b \sqrt{1 + [f'(x)]^2} dx$$



Find the length of the curve.

$$y = \frac{2}{3}(1+x^2)^{3/2} \quad 0 \leq x \leq 3$$



Find the length of the curve  $y = 1 + \frac{2}{3}(x-1)^{3/2}$  for  $1 \leq x \leq 4$ .



Find the length of the curve.

$$y = x^2 - \frac{\ln x}{8} \quad \text{for } 1 \leq x \leq 2.$$