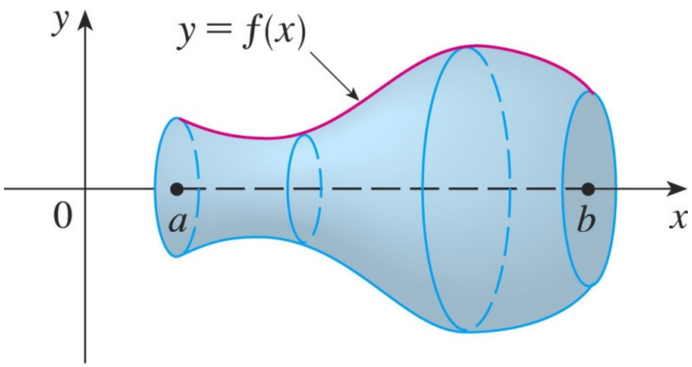
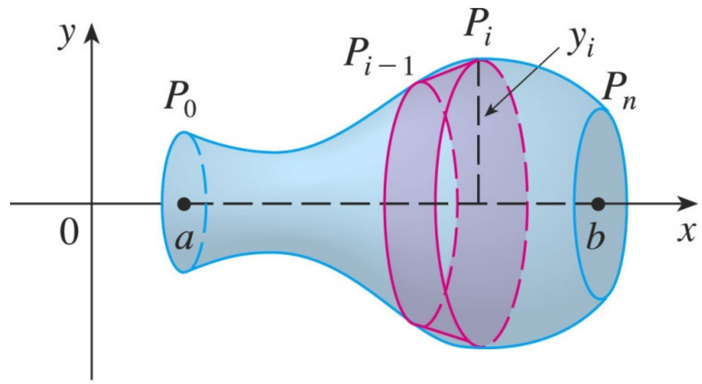


(a) Surface of revolution



(b) Approximating band

Area of the band $= 2\pi(\text{radius})(\text{length})$

For the radius, take an average.

For the length, take the distance from P_{i-1} to P_i.

$$= 2\pi\left(\frac{y_{i-1}+y_i}{2}\right)d\left(P_{i-1}P_i\right)$$

$$= 2\pi f\left(x_i^*\right)\sqrt{1+\left(f'\left(x_i^*\right)\right)^2}\,\Delta x$$

In 6.3 we saw $d\left(P_{i-1}P_i\right)=\sqrt{1+\left(f'\left(x_i^*\right)\right)^2}\,\Delta x$

For small Δx, $y_i = f\left(x_i\right) \approx f\left(x_i^*\right)$

and $y_{i-1} = f\left(x_{i-1}\right) \approx f\left(x_i^*\right)$

Total surface area $\approx \displaystyle\sum_{i=1}^{n} 2\pi f\left(x_i^*\right)\sqrt{1+\left(f'\left(x_i^*\right)\right)^2}\,\Delta x$

Total surface area (better approximation) $= \displaystyle\lim_{n\to\infty}\sum_{i=1}^{n} 2\pi f\left(x_i^*\right)\sqrt{1+\left(f'\left(x_i^*\right)\right)^2}\,\Delta x$

the area of surface obtained by rotating the curve $y = f\left(x\right)$ about the $x-$axis for $a \le x \le b$ is

$$\text{Surface Area} = \int_a^b 2\pi f\left(x\right)\sqrt{1+\left(f'\left(x\right)\right)^2}\,dx$$

A function with a continuous derivative on $[a,b]$ $\Rightarrow$ the area of surface obtained by rotating the graph of a function about the $y-\text{axis}$ for $a \le x \le b$ is

$$SA = 2\pi \int_a^b x\,ds$$

function: $y = f(x)$	function: $x = g(y)$ $\quad c \le y \le d$
$SA = 2\pi \int_a^b x\sqrt{1+\left[\frac{dy}{dx}\right]^2}\,dx$	$SA = 2\pi \int_c^d g(y)\sqrt{1+\left[\frac{dx}{dy}\right]^2}\,dy$

A function with a continuous derivative on $[a,b]$ $\Rightarrow$ the area of surface obtained by rotating the graph of a function about the $x-\text{axis}$ for $a \le x \le b$ is

$$SA = 2\pi \int_a^b y\,ds$$

function: $y = f(x)$	function: $x = g(y)$ $\quad c \le y \le d$
$SA = 2\pi \int_a^b f(x)\sqrt{1+\left[\frac{dy}{dx}\right]^2}\,dx$	$SA = 2\pi \int_c^d y\sqrt{1+\left[\frac{dx}{dy}\right]^2}\,dy$

Find the area of the surface formed by revolving the graph of $f(x) = x^2$ on the interval $0 \le x \le \sqrt{2}$ about the *y*-axis.

Find the area of the surface formed by revolving the graph of $x = \frac{1}{9}y^2 + 2$ on the interval $2 \le y \le 6$ about the x-axis.

Find the area of the surface formed by revolving the graph of $f(x)=\sqrt{x}$ on the interval $4 \le x \le 9$ about the *x*-axis.