8.2 Integrating Powers of Trig. Functions

1. $\int \cos^m x \sin^n x \, dx \quad (m, n \text{ positive integers})$

A) m, n : **one (or both) odd**

*ex.*1 one odd : $\int \cos^5 x \sin^2 x dx$

1. Factor out one power from the trig. function that has the odd power, if both have the odd power, just pick one of them and factor out one power.

2. use $\cos^2 x + \sin^2 x = 1$ to transform the remaining even power of the above trig function into the other trig. function

3. use u – substitution to finish the problem (let u = "other" trig function)

$\int \cos^m x \sin^n x \, dx \quad (m, n \text{ positive integers})$

$ex.2$ both odd : $\int \cos^3 x \sin^3 x dx$

1. Factor out one power from the trig. function that has the odd power, if both have the odd power, just pick one of them and factor out one power.

2. Use $\cos^2 x + \sin^2 x = 1$ to transform the remaining even power of the above trig function into the other trig. function

3. Use u − substitution to finish the problem (let u = "other" trig function)

$$\int \cos^m x \sin^n x \, dx \quad (m, n \text{ positive integers})$$

B) m, n: **both even** $\qquad ex: \int_0^{\pi} \cos^4 x \sin^2 x dx$

1. replace all even powers using the half-angle identities:

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ and $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$$\int_0^\pi \cos^4 x \sin^2 x dx = \frac{1}{8}\left[\underbrace{\int_0^\pi dx}_{A} + \underbrace{\int_0^\pi \cos 2x dx}_{B} - \underbrace{\int_0^\pi \cos^2 2x dx}_{C} - \underbrace{\int_0^\pi \cos^3 2x dx}_{D}\right]$$

$$A = \int_0^\pi dx =$$

$$B = \int_0^\pi \cos(2x)\, dx =$$

$$C = \int_0^\pi \cos^2 2x dx =$$

$$D = \int_0^\pi \cos^3 2x dx =$$

$$\int_0^\pi \cos^4 x \sin^2 x dx =$$

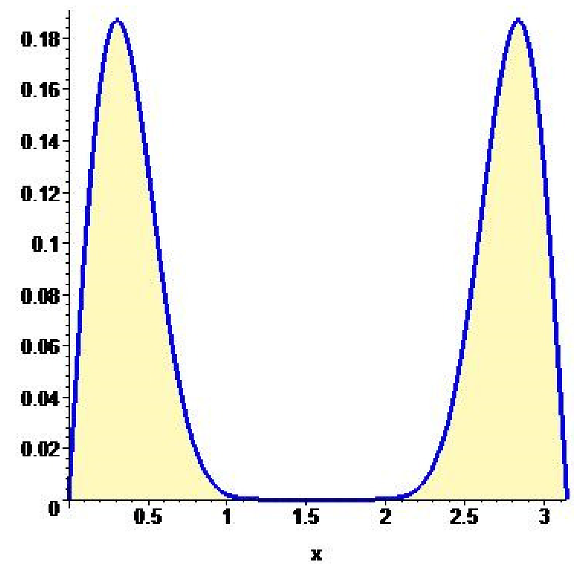
$$\int \cos^m x \sin^n x \, dx \quad (m, n \text{ positive integers})$$

C) m, n : **one or both = 1**

$$ex: \int_0^{\pi} \cos^{10} x \sin x dx$$

1. Just use $u-$substitution (let $u =$ the trig function with power $\neq 1$)

(if both $= 1$, choose either)



2. $\int \tan^m x \sec^n x \, dx \quad (m, n \text{ positive integers})$

A) m (the power of $\tan x$): **odd** $\qquad ex: \int \tan^3 x \sec^3 x dx$

1. Factor out one power of $\sec x$ and one power of $\tan x$

2. Use $\tan^2 x = \sec^2 x - 1$ to transform the remaining even power of $\tan x$ to be in terms of $\sec x$

3. Use $u-$substitution to finish the problem $(\text{let } u = \sec x)$

$\int \tan^m x \sec^n x \, dx \quad (m, n \text{ positive integers})$

B) n (the power of $\sec x$): **even** $\qquad ex: \int \tan^2 x \sec^4 x dx$

1. Factor out $\sec^2 x$

2. If $n > 2$, use $\sec^2 x = 1 + \tan^2 x$ to transform the remaining even power of $\sec x$ to be in terms of $\tan x$

3. use $u-$substitution to finish the problem $(\text{let } u = \tan x)$

$$\int \tan^m x \sec^n x \, dx$$

C) For all other cases, there is no set method ☹

Here are some examples:

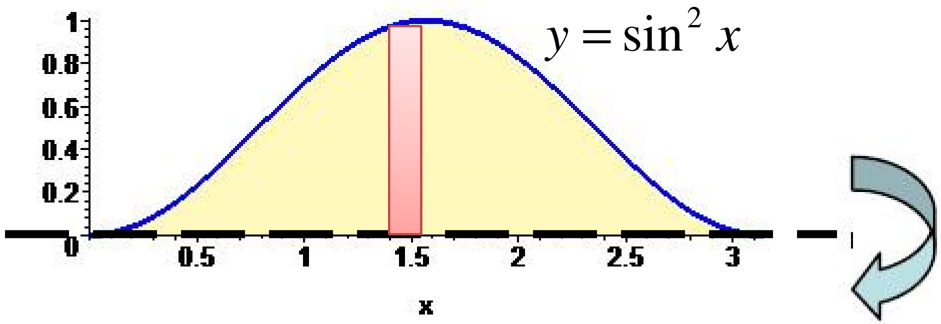
$ex: \int \tan x dx$

$ex: \int \sec x dx$

$ex:\int \tan^3 x dx$

$ex:\int \sec^3 x dx$

Find the volume of the solid obtained by rotating the region bounded by $y = \sin^2 x$ and $y = 0$ for $0 \le x \le \pi$ about the x-axis.



m, n integers with $m \neq n$

3. $\int \sin(mx)\sin(nx)\,dx \qquad \int \cos(mx)\cos(nx)\,dx \qquad \int \sin(mx)\cos(nx)\,dx$

We change the product into a sum using the following identities:

$$\sin(mx)\sin(nx) = \frac{1}{2}\left[\cos([m-n]x) - \cos([m+n]x)\right]$$

$$\cos(mx)\cos(nx) = \frac{1}{2}\left[\cos([m-n]x) + \cos([m+n]x)\right]$$

$$\sin(mx)\cos(nx) = \frac{1}{2}\left[\sin([m-n]x) + \sin([m+n]x)\right]$$