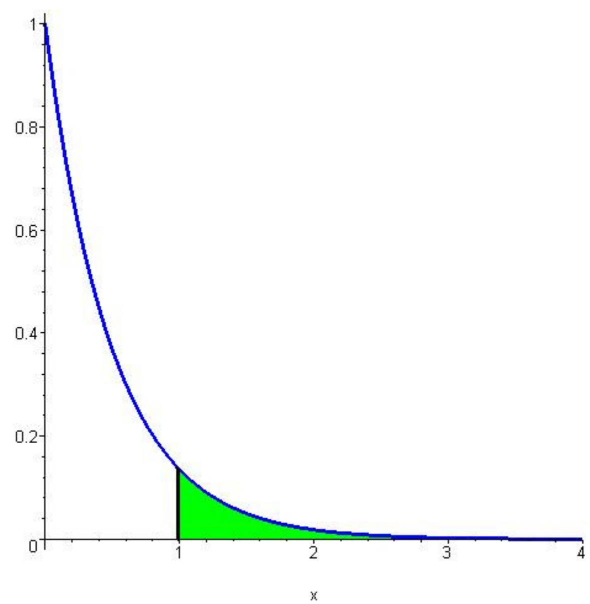
8.7 Improper Integrals

Infinite Upper Limit

$$\int_a^{\infty} f(x)\,dx = \lim_{t\to\infty}\int_a^{t} f(x)\,dx$$

$$\int_1^{\infty} e^{-2x}\,dx$$



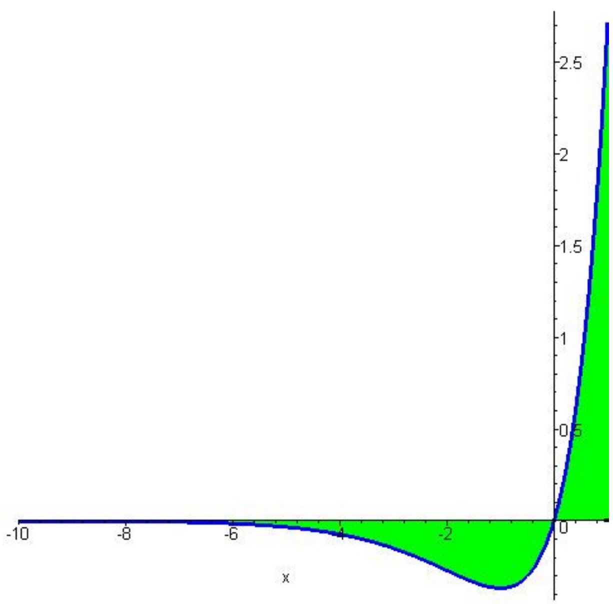
Infinite Upper Limit

$$\int_{1}^{\infty} \frac{e^{x}}{1+e^{x}}\,dx$$

Infinite Lower Limit

$$\int_{-\infty}^{b} f(x)\,dx = \lim_{a\to-\infty}\int_{a}^{b} f(x)\,dx$$

$$\int_{-\infty}^{1} xe^{x}\,dx$$

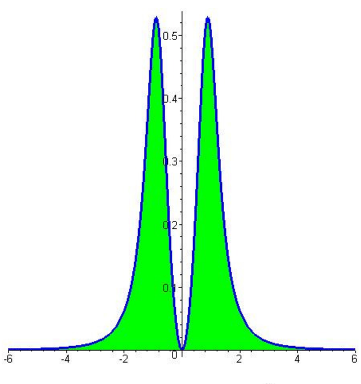


Infinite Upper and Lower Limit

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{c} f(x)\,dx + \int_{c}^{\infty} f(x)\,dx \;;c \text{ any real number}$$

$$\int_{-\infty}^{\infty} f(x)\,dx = \lim_{a\to-\infty}\int_{a}^{c} f(x)\,dx + \lim_{b\to\infty}\int_{c}^{b} f(x)\,dx$$

$$\int_{-\infty}^{\infty} \frac{x^2}{1+x^6}\,dx$$



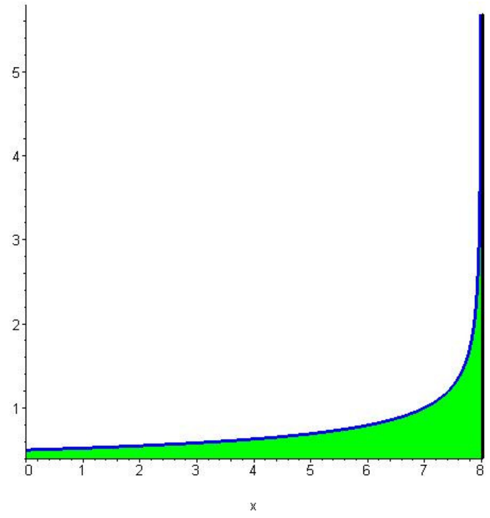
Infinite Discontinuity at Upper Limit

$$\int_a^b f(x)\,dx = \lim_{t\to b^-}\int_a^t f(x)\,dx$$

$f(b) \to$ infinite discontinuity

$$\int_0^8 \frac{dx}{\sqrt[3]{8-x}}$$

$f(8) \to$ infinite discontinuity



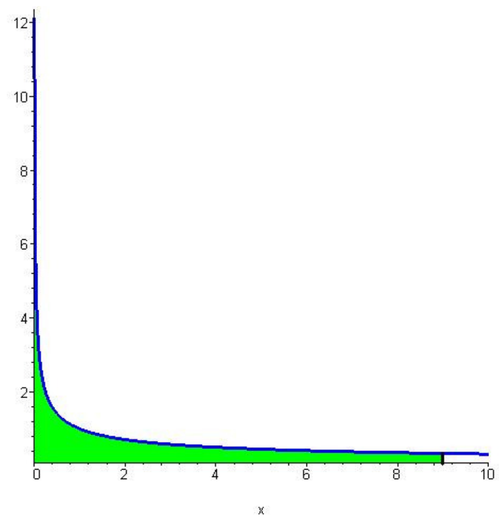
Infinite Discontinuity at Lower Limit

$$\int_a^b f(x)\,dx = \lim_{t\to a^+} \int_t^b f(x)\,dx$$

$f(a)\to$infinite discontinuity

$$\int_0^9 \frac{dx}{\sqrt{x}}$$

$f(0)\to$infinite discontinuity



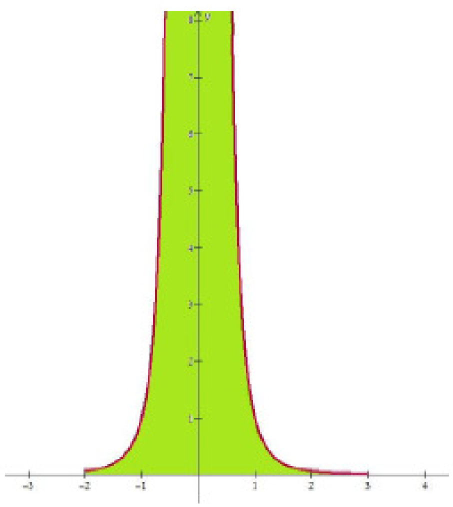
Infinite Discontinuity inside the interval

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \lim_{t\to c^-}\int_a^t f(x)\,dx + \lim_{t\to c^+}\int_t^b f(x)\,dx$$

$f(c)\to$ infinite discontinuity
$a<c<b$

$$\int_{-2}^{3}\frac{dx}{x^4}$$

$f(0)\to$ infinite discontinuity
-2<0<3



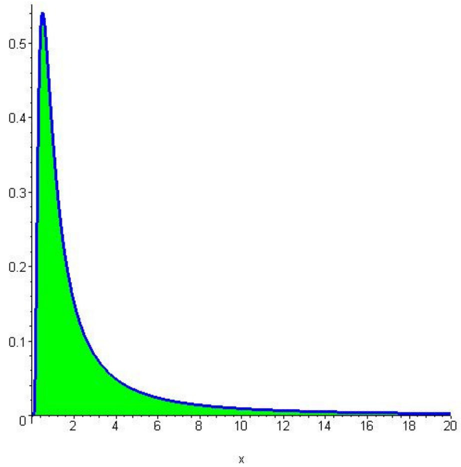
Doubly Improper

$$\int_0^\infty f(x)\,dx = \int_0^c f(x)\,dx + \int_c^\infty f(x)\,dx = \lim_{a\to 0^+}\int_a^c f(x)\,dx + \lim_{b\to\infty}\int_c^b f(x)\,dx$$

$f(0) \to$ infinite discontinuity

$$\int_0^\infty \frac{e^{-1/x}}{x^2}\,dx$$

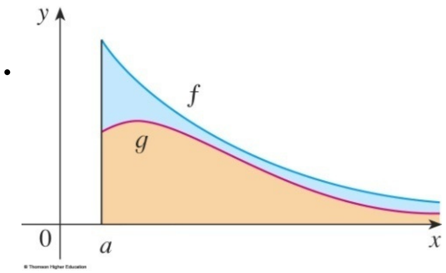
$f(0) \to$ infinite discontinuity



Direct Comparison Theorem

Suppose that $f(x)$ and $g(x)$ are continuous functions with $f(x) \geq g(x) \geq 0$ for $x \geq a$.

$a)$ If $\int_a^\infty f(x)\,dx$ is ____________, then $\int_a^\infty g(x)\,dx$ is ____________.



If your function is **smaller** than a function with a ______ area, then your function will also have ________ area

$b)$ If $\int_a^\infty g(x)\,dx$ is _________, then $\int_a^\infty f(x)\,dx$ is ____________.

If your function is **larger** than a function with __________ **area**, then your function will also have ____________ area.

Two examples worked on the next slides: $\int_1^\infty \frac{x}{x^3+1}\,dx$ $\int_1^\infty \frac{2+e^{-x}}{x}\,dx$

Two examples worked out on my YouTube videos:

$\int_2^\infty \frac{x+1}{\sqrt{x^4-x}}\,dx$ $\int_0^\infty \frac{\arctan x}{2+e^x}\,dx$

http://youtu.be/ZYMIAlFeDvc http://youtu.be/FzXKyf0_b0c

$$\int_1^{\infty} \frac{x}{x^3+1}\,dx$$

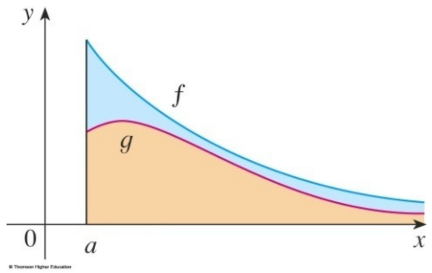
$$\int_1^{\infty} \frac{2+e^{-x}}{x}\,dx$$

What do you do if the inequality goes the wrong direction?

$$\int_2^\infty \frac{x}{\sqrt{x^4+3x}}\,dx$$

So your function is smaller than a function that has infinite area

For the direct comparison to work your function needs to be larger than the one with infinite area or smaller than one with finite area.



The good news is that you can still recover for some cases using:

Limit Comparison Theorem

Suppose that $f(x)$ and $g(x)$ are continuous positive functions for $x \ge a$,

and $\lim_{x\to\infty} \frac{f(x)}{g(x)} = L$ with $0 < L < \infty$,

then ________________________________.

$$\int_2^{\infty} \frac{x}{\sqrt{x^4+3x}}\,dx$$

$$\int_{1}^{\infty} \frac{1}{\sqrt{e^x - x}}\,dx$$