

## 9.2 Linear Differential Equation

Math 104 – Rimmer  
9.2 Linear Diff. Eq.

$$\frac{dy}{dx} + P(x)y = Q(x) \leftarrow \text{standard form of a linear first order diff. eq.}$$

1) Find the integrating factor  $\mu = e^{\int P(x)dx}$

2) Multiply the entire equation by it.

$$e^{\int P(x)dx} \frac{dy}{dx} + P(x)e^{\int P(x)dx} y = e^{\int P(x)dx} Q(x)$$

3) Identify the left hand side as the product rule with the two functions being the integrating factor and the unknown function  $y(x)$ .

$$\underbrace{e^{\int P(x)dx} \frac{dy}{dx} + P(x)e^{\int P(x)dx} y}_{\frac{d}{dx} \left( e^{\int P(x)dx} \cdot y \right)} = e^{\int P(x)dx} Q(x)$$

4) Integrate both sides of the equation.

$$\int \left[ \frac{d}{dx} \left( e^{\int P(x)dx} \cdot y \right) \right] dx = \int \left[ e^{\int P(x)dx} Q(x) \right] dx$$

5) Use the FTC on the left hand side.

$$e^{\int P(x)dx} \cdot y = \int \left[ e^{\int P(x)dx} Q(x) \right] dx$$

6) Divide both sides by  $\mu$ .

$$y = \frac{\int \left[ e^{\int P(x)dx} Q(x) \right] dx}{e^{\int P(x)dx}}$$

$$x^2 y' + 2xy = \cos^2 x$$

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$$x \ln x \frac{dy}{dx} + y = xe^x$$


Let  $y(t)$  be the solution to the differential equation

$$t \frac{dy}{dt} = t^2 + 2y$$


satisfying  $y(1) = 2$ . What is  $y(2)$ ?

- (A)  $2 + 4 \ln 2$       (B)  $8 + 4 \ln 2$       (C)  $4 + 2 \ln 2$       (D)  $8 + 8 \ln 2$   
(E)  $2 + 8 \ln 2$       (F)  $4 + 4 \ln 2$       (G)  $2 + 2 \ln 2$       (H)  $4 + 8 \ln 2$

$\frac{dy}{dt} = -k(y+1)$  with  $y(0) = 2$  and  $y(1) = 1$ . Find  $y(2)$ .



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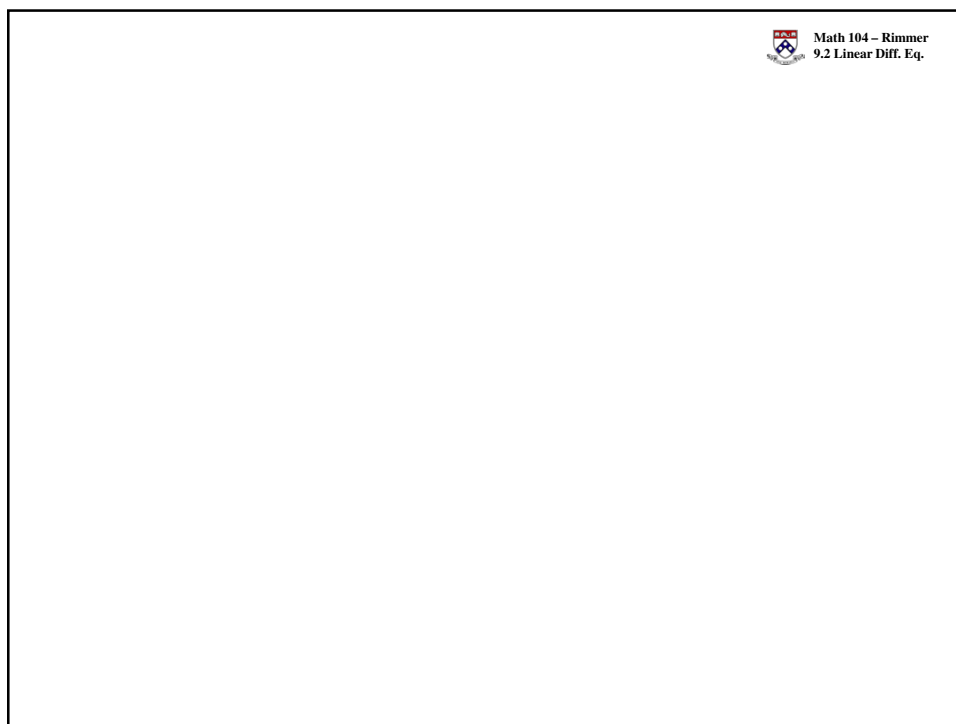


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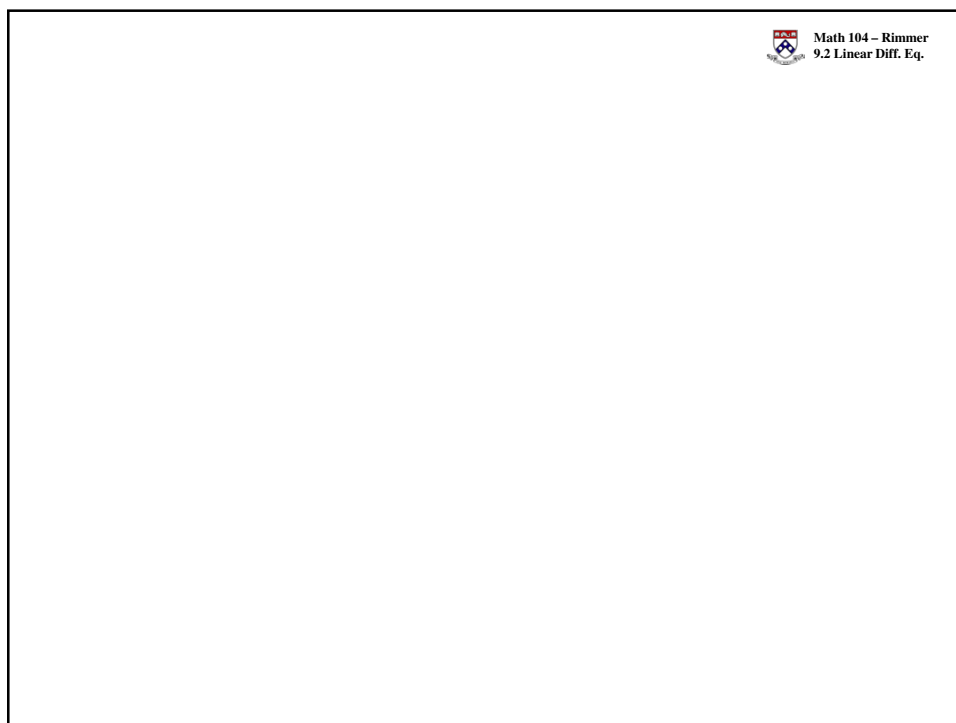
In Section **7.2** we looked at mixing problems in which the volume of fluid remained constant and saw that such problems give rise to separable equations.

If the rates of flow into and out of the system are different, then the volume is not constant and the resulting differential equation is linear but not separable.

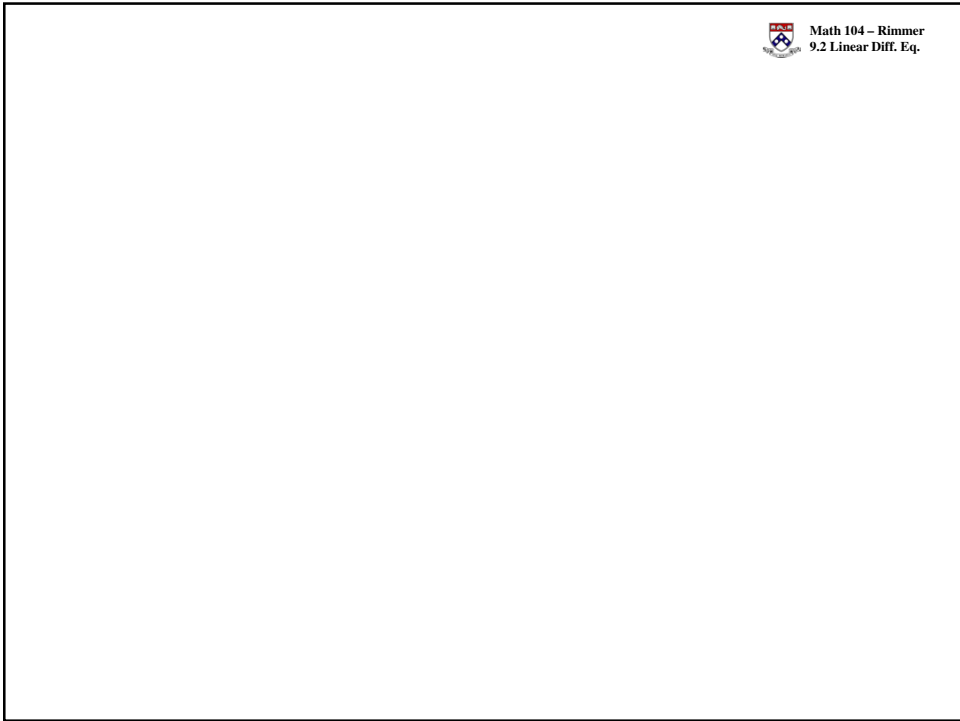
A tank contains 100 L of water. A solution with a salt concentration of 0.4 kg/L is added at a rate of 5 L/min. The solution is kept mixed and is drained from the tank at a rate of 3 L/min. If  $y(t)$  is the amount of salt (in kilograms) after  $t$  minutes, **find**  $y(t)$



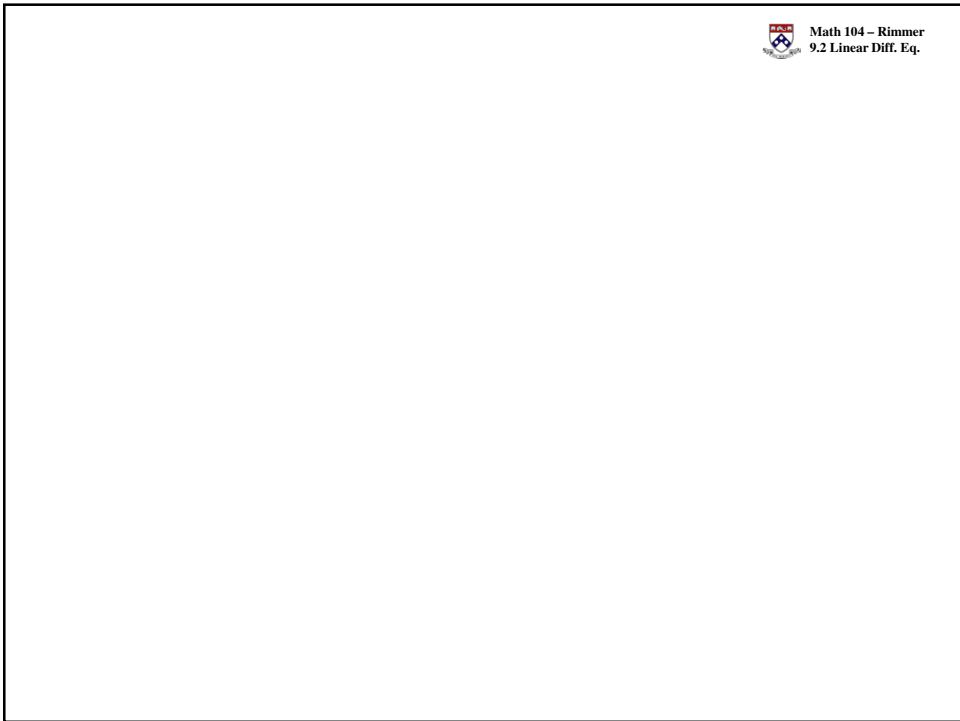
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