12.4 Cross Product

The **cross product** of two vectors is a ______ with the special quality of being ____________ to both original vectors.

The cross product yields a ____ in contrast to the dot product that yields a _______________

The **cross product** of $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ is

$$\mathbf{u} \times \mathbf{v} = \langle u_2 v_3 - u_3 v_2, u_3 v_1 - u_1 v_3, u_1 v_2 - u_2 v_1 \rangle$$

The definition ____.

(The cross product is ____ defined for two-dimensional vectors.)

Instead of memorizing what gets multiplied by what, there is a convenient way to calculate $\mathbf{u} \times \mathbf{v}$ using the __________ form with ____________________.

Determinant

$\underline{2 \times 2}$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} =$$

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} =$$

$$\begin{vmatrix} -6 & 2 \\ -9 & 3 \end{vmatrix} =$$

$$\begin{vmatrix} 0 & 3 \\ -1 & 99 \end{vmatrix} =$$

3X3 Determinants

Reduces to finding __ 2×2 determinants using cofactor expansion on the ________.
Take each entry in the first row, we will multiply each of these entries by a 2×2 determinant.

The 2×2 determinants are found by __________ that entry's column and row.

One last thing is to ________________________.

$$\begin{vmatrix} 1 & -3 & 2 \\ -1 & 9 & 4 \\ -5 & 3 & 1 \end{vmatrix} =$$

<u>3×3 Shortcut</u>

Copy 1st 2 columns

$$\begin{vmatrix} 1 & 6 & -2 \\ 3 & -1 & 3 \\ 4 & 5 & 2 \end{vmatrix} \begin{matrix} 1 & 6 \\ 3 & -1 \\ 4 & 5 \end{matrix} = \begin{pmatrix} \text{sum of} \\ \text{forward} \\ \text{diagonal} \\ \text{products} \end{pmatrix} - \begin{pmatrix} \text{sum of} \\ \text{backward} \\ \text{diagonal} \\ \text{products} \end{pmatrix}$$

How to find the cross product using determinants

$$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} =$$

$$\mathbf{u}\times\mathbf{v} = (u_2v_3 - u_3v_2)\mathbf{i} - (u_1v_3 - u_3v_1)\mathbf{j} + (u_1v_2 - u_2v_1)\mathbf{k}$$

$$\mathbf{u}\times\mathbf{v} = \langle u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1 \rangle$$

Let $\mathbf{u} = \langle 1,-2,1\rangle$ and $\mathbf{v} = \langle 3,1,-2\rangle$ Find $\mathbf{u}\times\mathbf{v}$.

$$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 1 \\ 3 & 1 & -2 \end{vmatrix} =$$

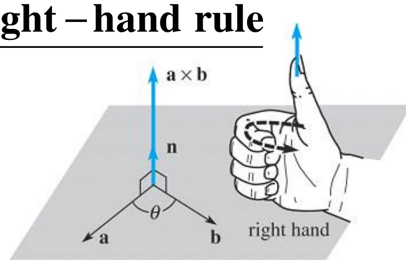
Let $\mathbf{u} = \langle 1,1,1 \rangle$ and $\mathbf{v} = \langle 2,1,-1 \rangle$ Find $\mathbf{u} \times \mathbf{v}$ and show that it is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.

Algebraic Properties of the cross product:

Let $\mathbf{u}, \mathbf{v},$ and $\mathbf{w}$ be vectors and let c be a scalar.

1. $\mathbf{u} \times \mathbf{v} =$
2. $\mathbf{u} \times (\mathbf{v} + \mathbf{w}) =$
3. $c(\mathbf{u} \times \mathbf{v}) =$
4. $\mathbf{0} \times \mathbf{v} =$
5. $\mathbf{v} \times \mathbf{v} =$
6. $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) =$
7. $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) =$

Right – hand rule



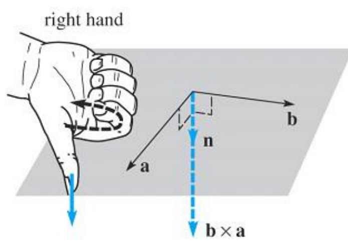
Place your 4 fingers in the direction of the __________,

___ them in the direction of the _______________,

Your ______ will point in the direction of the cross product

$$\mathbf{u}\times\mathbf{v} = -(\mathbf{v}\times\mathbf{u})$$

(by switching the order, you get a vector __________________)



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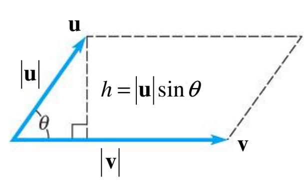
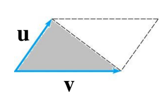
12.4 Cross Product

Geometric Properties of the cross product:

Let $\mathbf{u}$ and $\mathbf{v}$ be nonzero vectors and let θ be the angle between $\mathbf{u}$ and $\mathbf{v}$.

1. $\mathbf{u}\times\mathbf{v}$ is _____________ to both $\mathbf{u}$ and $\mathbf{v}$.
2. $|\mathbf{u}\times\mathbf{v}| =$
3. $\mathbf{u}\times\mathbf{v} = \mathbf{0}$ if and only if
4. $|\mathbf{u}\times\mathbf{v}| =$
5. $\frac{1}{2}|\mathbf{u}\times\mathbf{v}| =$

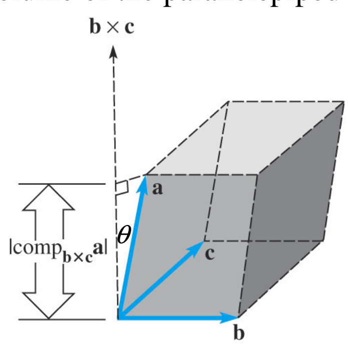




A nice online java applet for the cross product can be found here:

http://www.phy.syr.edu/courses/java-suite/crosspro.html

Volume of the parallelepiped determined by the vectors $\mathbf{a}, \mathbf{b},$ and $\mathbf{c}$.



Area of the base =

Height =

Volume =

Volume =

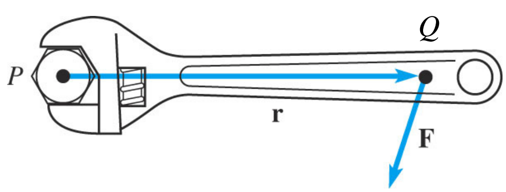
$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$ is called the ______________

The vectors are in the same plane (_______) if the scalar triple product ____.

The scalar triple product can be written as a determinant:

Let $\mathbf{u} = \langle 2,0,1 \rangle$, $\mathbf{v} = \langle 1,1,1 \rangle$ and $\mathbf{w} = \langle 0,2,2 \rangle$. Find $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$.

In physics, the cross product is used to measure ______.

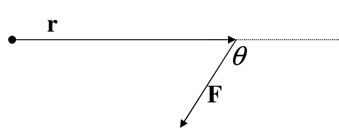


Consider a force $\mathbf{F}$ acting on a rigid body at a point given by a position vector $\mathbf{r}$.

The ______ (τ) measures the tendency of the body to ______ about the origin (point P)

$$\tau =$$

$$|\tau| =$$



(θ is the angle between the _____ and __________ vectors)