Moduli Stabilization in Heterotic Theories

Lara B. Anderson

Department of Physics, University of Pennsylvania

Work done in collaboration with:

LBA, J. Gray, A. Lukas, B. Ovrut

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A heterotic model

We begin with the $E_8 \times E_8$ Heterotic string in 10-dimensions:

- The geometric ingredients include:
  - A Calabi-Yau 3-fold, X
  - A holomorphic vector bundle, V, on X (with structure group $G \subset E_8$)
- Compactifying on X leads to $\mathcal{N} = 1$ SUSY in $4D$, while V breaks $E_8 \to G \times H$, where H is the Low Energy GUT group
  - $G = SU(n)$, $n = 3, 4, 5$ leads to $H = E_6, SO(10), SU(5)$
- Matter and Moduli
  - H-charged matter, $H^1(X, V)$, $H^1(X, V^\vee)$, $H^1(X, \wedge^2 V)$, ...
  - $X \Rightarrow h^{1,1}(X)$ - Kähler moduli and $h^{2,1}(X)$ - Complex structure moduli
  - $V \Rightarrow h^1(X, V \times V^\vee)$ Bundle moduli
- Numerous models known with MSSM spectrum, but need moduli stabilization

Supersymmetric Vacua in Heterotic Theories

A supersymmetric vacuum to the theory must satsify the Hermitian Yang-Mills Equations

- $\delta\chi = 0 \Rightarrow \begin{cases} F_{ab} = F_{\bar{a}\bar{b}} = 0 \\ g^{a\bar{b}} F_{a\bar{b}} = 0 \end{cases}$
- Solution depends on complex structure, Kähler and bundle moduli. Some regions of moduli space will provide a solution, some not.
- In 10D: $S_{partial} \sim \int_{M_{10}} Tr(F^{(1)})^2 + Tr(F^{(2)})^2 - Tr(R^2) + \dots$
- Leads to: $S_{partial} \sim \int_{M_{10}} \sqrt{-g}\{(F^{(1)}{}_{a\bar{b}} g^{a\bar{b}})^2 + (F^{(2)}{}_{a\bar{b}} g^{a\bar{b}})^2 + (F^{(1)}{}_{ab} F^{(1)}{}_{\bar{a}\bar{b}} g^{a\bar{a}} g^{a\bar{a}}) + (F^{(2)}{}_{ab} F^{(2)}{}_{\bar{a}\bar{b}} g^{a\bar{a}} g^{a\bar{a}})\} + \dots$
- Contributes to the $4D$ potential. Don't know $F_{a\bar{b}}$, F_{ab} and $g^{a\bar{b}}$ except numerically.

Holomorphic Vector bundles

- In this talk, we'll look at $F_{ab} = 0$
- Recall, a vector bundle is said to be holomorphic if $F_{ab} = F_{\bar{a}\bar{b}} = 0$
- Suppose we begin with a holomorphic bundle w.r.t a fixed complex structure. What happens as we vary the complex structure? Must a bundle stay holomorphic for any variation $\delta \mathfrak{z}^I v_I \in h^{2,1}(X)$? $\Rightarrow$ No.
- In real coordinates we introduce the projectors

$$P_\mu^{\ \nu} = (\mathbb{1}_\mu^{\ \nu} + i\mathcal{J}_\mu^{\ \nu}) \quad \bar{P}_\mu^{\ \nu} = (\mathbb{1}_\mu^{\ \nu} - i\mathcal{J}_\mu^{\ \nu}) \tag{1}$$

Where $\mathcal{J}^2 = -\mathbb{1}$ is the complex structure tensor. Leads to

$$g^{\mu\nu} P_\mu^\gamma \bar{P}_\nu^\delta F_{\gamma\delta} \quad = \quad 0 \tag{2}$$

$$P_\mu^{\ \nu} P_\rho^{\ \sigma} F_{\nu\sigma} = 0 \quad , \quad \bar{P}_\mu^{\ \nu} \bar{P}_\rho^{\ \sigma} F_{\nu\sigma} = 0 \tag{3}$$

Varying the complex structure

- Consider change in $F_{ab} = 0$ under the perturbation

$$\mathcal{J} = \mathcal{J}^{(0)} + \delta\mathcal{J} \qquad A = A^{(0)} + \delta A \tag{4}$$

$\delta\mathcal{J} \to \delta P$

- In the original coords, to first order this leads to

$$\delta\mathfrak{z}^I v^c_{I[\bar{a}} F^{(0)}_{|c|\bar{b}]} + 2D^{(0)}_{[\bar{a}} \delta A_{\bar{b}]} = 0 \tag{5}$$

- Rotation of $F^{1,1}$ into $F^{0,2}$ plus change in $F^{0,2}$ due to change in gauge connection.
- Question: For each $\delta\mathfrak{z}^I$ is there a δA which compensates?
- Answer: Not in general.
- The Central Idea: Use bundle holomorphy to constrain C.S. Moduli

Deformation Theory

There are three objects in deformation theory that we need

- $Def(X)$: Deformations of X as a complex manifold. Infinitesimal defs parameterized by the vector space $H^1(TX) = H^{2,1}(X)$. These are the *complex structure* deformations of X.
- $Def(V)$: The deformation space of V (changes in connection, δA) *for fixed* C.S. moduli. Infinitesimal defs measured by $H^1(End(V)) = H^1(V \otimes V^\vee)$. These define the *bundle moduli* of V.
- $Def(V, X)$: Simultaneous holomorphic deformations of V and X. The tangent space is $H^1(X, \mathcal{Q})$ where

$$0 \to V \otimes V^\vee \to \mathcal{Q} \xrightarrow{\pi} TX \to 0 \tag{6}$$

  $\mathcal{Q}$ is defined by the projectivized total space of the bundle $\mathbb{P}(V) \to X$.
- $H^1(X, \mathcal{Q})$ are the real moduli of a heterotic theory!

The Atiyah Sequence

- $0 \to V \otimes V^{\vee} \to \mathcal{Q} \xrightarrow{\pi} TX \to 0$ is known as the Atiyah sequence.
- The long exact sequence in cohomology gives us

$$0 \to H^1(V \otimes V^{\vee}) \to H^1(\mathcal{Q}) \xrightarrow{d\pi} H^1(TX) \xrightarrow{\alpha} H^2(V \otimes V^{\vee}) \to \dots \qquad (7)$$

- $H^1(\mathcal{Q}) = H^1(V \otimes V^{\vee}) \oplus Im(d\pi)$. But $d\pi$ not surjective in general!
- By exactness, $Im(d\pi) = Ker(\alpha)$ where

$$\alpha = [F^{1,1}] \in H^1(V \otimes V^{\vee} \otimes TX^{\vee}) \qquad (8)$$

  is the Atiyah Class
- C.S. moduli allowed $\alpha(\delta_{\mathfrak{z}} v) = 0 \quad (0 \in H^2(V \times V^{\vee}))$. I.e. in $Ker(\alpha)$

$$\delta_{\mathfrak{z}}{}^I v^c_{I[\bar{a}} F_{|c|\bar{b}]} = D_{[\bar{a}} \Lambda_{\bar{b}]} \;\; (= 0 \in H^2(V \times V^{\vee})) \qquad (9)$$

- Now, if we let $\Lambda = -2\delta A$ we recover

$$\delta \mathfrak{z}^I v^c_{I[\bar{a}} F^{(0)}_{|c|\bar{b}]} + 2D^{(0)}_{[\bar{a}} \delta A_{\bar{b}]} = 0 \tag{10}$$

- That is, the fluctuation of the $10d$ E.O.M. $F_{ab} = 0$ is implied by the Atiyah sequence.
- Note that the bundle moduli are unaffected. I.e. an injection $0 \to H^1(V \otimes V^\vee) \to H^1(\mathcal{Q})$.
- We want to know:
  - $Ker(\alpha)$: Free C.S. moduli
  - $Im(\alpha)$: Stabilized C.S. moduli
- Using computational algebraic geometry, this is hard, but can be done!
- Question: What is $Im(\alpha)$? How many moduli fixed for a given bundle?

A Threefold Example

- Start simple...
- An extension: $0 \to \mathcal{L} \to V \to \mathcal{L}^\vee \to 0$
- For example on the Calabi-Yau threefold $X = \left[\begin{array}{c|c} \mathbb{P}^2 & 3 \\ \mathbb{P}^2 & 3 \end{array} \right]^{2,83}$

$$0 \to \mathcal{O}(-3,3) \to V \to \mathcal{O}(3,-3) \to 0 \tag{11}$$

- Why this one? Here $Ext^1(\mathcal{L}^\vee, \mathcal{L}) = H^1(X, \mathcal{O}(-6,6)) = 0$ generically. Hence, cannot define the bundle for general complex structure!
- Happily, cohomology can "jump" at higher co-dimensional loci in C.S. moduli space.
- We can explicitly solve for when $H^1(X, \mathcal{L}^2) \neq 0$ and we find that on a 2-dimensional locus in C.S. moduli space, $h^1(X, \mathcal{O}(-6,6)) = 180$.
- Aside: Known Heterotic Standard Models on this manifold.

Jumping cohomology and the Atiyah class

- Since this extension bundle cannot be defined away from this 2-dimensional locus we expect $im(\alpha) \geq 81$.
  - Let $\mathcal{A} = \mathbb{P}^2 \times \mathbb{P}^2$. The Koszul sequence for X gives us
    $0 \to \mathcal{O}(-3,-3) \otimes \mathcal{L}_{\mathcal{A}} \stackrel{p_0}{\to} \mathcal{L}_{\mathcal{A}} \to \mathcal{L}_X \to 0$
    $H^1(X, \mathcal{O}(-6,6)) = ker(p_0)$, $p_0 : H^2(\mathcal{A}, \mathcal{O}(-9,3)) \stackrel{p_0}{\to} H^2(\mathcal{A}, \mathcal{O}(-6,6))$

  Vary p_0 so that $ker(p_0) \neq 0$.
- This "jumping" substructure is incredibly rich. Hundreds of disconnected higher co-dimensional loci for one even one line bundle: $h^1(\mathcal{O}(-6,6)) = 12, 32, 98, 180....$
- Also: Begin at a point, p_0 for which $Ext \neq 0$, do Atiyah computation of linear deformations.
- Have explicitly generated polynomial basis of source, target and map for $H^1(TX) \stackrel{\alpha}{\to} H^2(V \otimes V^\vee) \Rightarrow Im(\alpha) = 81$ (No. of moduli stabilized)

4D Field Theory

- For the $4d$ Theory: We have the Gukov-Vafa-Witten superpotential $W = \int_X \Omega \wedge H$ where $H = dB - \frac{3\alpha'}{\sqrt{2}}\left(\omega^{3\text{YM}} - \omega^{3\text{L}}\right)$
- In Minkowski vacuum (with $W = 0$), F-terms: $F_{C_i} = \frac{\partial W}{\partial C_i} = -\frac{3\alpha'}{\sqrt{2}} \int_X \Omega \wedge \frac{\partial \omega^{3\text{YM}}}{\partial C_i}$
- Dimensional Reduction Anzatz: $A_\mu = A_\mu^{(0)} + \delta A_\mu + \bar{\omega}_\mu^i \delta C_i + \omega_\mu^i \delta \bar{C}_i$

$$\delta(F_{C_i}) = \int_X \epsilon^{\bar{a}\bar{c}\bar{b}} \epsilon^{abc} \Omega^{(0)}_{abc} 2\bar{\omega}^{xi}_{\bar{c}} \, \text{tr}(T_x T_y) \left(\delta \mathfrak{z}^I v^c_{I[\bar{a}} F^{(0)y}_{|c|\bar{b}]} + 2D^{(0)}_{[\bar{a}} \delta A^y_{\bar{b}]}\right)$$

- In general, $\mathfrak{z}$ is stabilized at the compactification scale. To explicitly describe F-terms F_{C_i}, we must find a region of moduli space for which $\mathfrak{z}$ is light.
- Stabilize C.S. moduli perturbatively in a supersymmetric Minkowski vacuum. Topologically trivial flux → Still a CY manifold.

Three Equivalent Approaches

Three ways to determine/engineer C.S. Stabilization:

1. Atiyah Class Computation: Directly compute $Im(\alpha)$,
   $\alpha : H^1(TX) \to H^2(V \times V^\vee)$
2. Field Theory (solve F-terms)
3. Study "Jumping" of key bundle support.

- Intuitively, we expect these three things to give the same answer. And in fact, we prove that these views are equivalent in a broad class of examples.
- No.3 is the easiest. And much progress can be made looking at examples whose very definition clearly varies with complex structure.
- Can be done for Extension, Monad, Spectral Cover, Serre and Maruyama Constructions. For each, simple "structural" failures of holomorphy can be found.

Conclusions – Complex Structure Moduli

- The presence of a *holomorphic* vector bundle constrains C.S. moduli
- The moduli of a heterotic compactification: $H^{1,1}(X)$, $H^1(V \otimes V^\vee)$, $Ker(\alpha)$
- $Im(\alpha)$ can be computed
- Leads to F-terms in 4-dimensions: $\frac{\partial W}{\partial C_I}$ where C_I are $4d$ matter fields
- The C.S. can be stabilized at the perturbative level without moving away from a CY manifold
  - Avoids problems of naive KKLT scenarios in heterotic
  - Allows us to keep the toolkit of Kähler geometry (model-building)
  - Generic: For all known classes of CY manifolds it is straightforward to produce simple, consistent bundles to stabilize C.S moduli.
- Provides a general Hidden Sector mechanism for stabilizing the C.S. moduli in Heterotic (M-theory) compactifications.

The End