Heterotic/F-theory duality and lattice polarized K3's

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Andreas Malmendier, Colby College
(joint work with David Morrison)

Heterotic/F-theory Duality

The heterotic string comactified on an $(n-1)$-dimensional elliptically fibered Calabi-Yau $\pi_H : \mathbf{Z} \to \mathbf{B}$ is equivalent to F-theory compactified on an n-dimensional K3-fibered Calabi-Yau $\pi_F : \mathbf{X} \to \mathbf{B}$, which is also elliptically fibered with a section.

Eight-dimensional compactifications: $n = 2$ and $\mathbf{B} = \mathrm{pt}$

- Heterotic CY: $\mathbf{Z} = E$ elliptic curve w/ principal G-bundle, $G = (E_8 \times E_8) \rtimes \mathbb{Z}_2$ or $\mathrm{Spin}(32)/\mathbb{Z}_2$.
- F-theoretic CY: elliptic K3-surface $\mathbf{X} \to \mathbb{CP}^1$ w/ section,
  $$\bar{\mathbf{X}} : \; Y^2 = 4\,X^3 - g_2\,X - g_3\,, \qquad g_2 \in H^0(\mathcal{O}(8)),\; g_3 \in H^0(\mathcal{O}(12)).$$
- Moduli spaces for both types are given by the Narain space
  $$\mathfrak{M} = \mathrm{SO}(2,18;\mathbb{Z})\Big\backslash \mathrm{SO}(2,18)\Big/\Big(\mathrm{SO}(2) \times \mathrm{SO}(18)\Big)\,.$$
- Question: Can we describe duality map w/o taking limits?

F-theoretic description of type IIB string backgrounds

- Singular fiber where $\Delta = g_2^3 - 27\, g_3^2$ vanishes.
- Kodaira's classification of singular fibers:

	$\text{ord}_D(g_2)$	$\text{ord}_D(g_3)$	$\text{ord}_D(\Delta)$	singularity	monodromy
$I_n,\ n \geq 1$	0	0	n	A_{n-1}	$\begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$
$I_n^*,\ n \geq 0$	2	3	$n+6$	D_{n+4}	$\begin{pmatrix} -1 & n \\ 0 & -1 \end{pmatrix}$
III^*	3	≥ 5	9	E_7	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
II^*	≥ 4	5	10	E_8	$\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$
...	...	...	...	...	...

- Correspondence: string perspective $\leftrightarrow$ monodromy $+\ j = \frac{g_2^3}{\Delta}$

Heterotic string backgrounds

- Closed string theory on T^2 has two basic moduli:
  1) complex structure parameter $\tau \in \mathbb{H}$,
  2) complexified Kähler modulus $\rho = B + i\,V \in \mathbb{H}$.
- **Geometric** compactifications:
  τ varies over base, undergoes monodromies in $\mathrm{SL}(2,\mathbb{Z})$,
  ρ is constant up to shifts.
- **Quantum** compactifications: τ and ρ vary over base,
  $\rho \to -1/\rho$ possible, inherently quantum.
- Moduli of heterotic string compactified on T^2 near boundary:

$$\underbrace{\begin{pmatrix}\text{complex str.}\\ \text{param.}\end{pmatrix}}_{\tau \circlearrowleft \mathrm{SL}(2,\mathbb{Z})} \times \underbrace{\begin{pmatrix}\text{Kähler}\\ \text{param.}\end{pmatrix}}_{\rho \circlearrowleft \mathrm{SL}(2,\mathbb{Z})} \times \underbrace{\begin{pmatrix}\text{Wilson lines}\end{pmatrix}}_{z,\dots}$$

Matching the moduli: F-theory $\leftrightarrow$ heterotic string

- Our approach: compactifications w/ very few Wilson lines

$$\mathrm{SO}(2,r;\mathbb{Z})\Big\backslash \mathrm{SO}(2,r)\Big/\Big(\mathrm{SO}(2)\times\mathrm{SO}(r)\Big) \subset \mathfrak{M}\,.$$

r=2: No Wilson lines. $G=(E_8\times E_8)\rtimes\mathbb{Z}_2$ or $G=\mathrm{Spin}(32)/\mathbb{Z}_2$.
r=3: One Wilson line. $G=E_8\times E_7$ or $G=\mathrm{Spin}(28)\times\mathrm{SU}(2)/\mathbb{Z}_2$.

- Use (Siegel) modular forms and Shioda-Inose correspondence to describe moduli spaces and the duality map.
- Use duality map to extend description of heterotic backgrounds to include quantum compactifications.

Weierstrass fibrations → lattice polarized K3's

$$\bar{\mathbf{X}} \to \mathbb{CP}^1 : \quad \left(\begin{array}{ccl} Y^2 & = & 4\,X^3 + \left(a\,u^4 + c\,u^3\right) X \\ & + & \left(u^7 + b\,u^6 + d\,u^5\right) \end{array} \right)$$

$c = 0$: No Wilson lines. Gauge group $G = (E_8 \times E_8) \rtimes \mathbb{Z}_2$.

$$\begin{array}{lcll} \text{sing. fibers of } \bar{\mathbf{X}} & : & 2\,II^* \oplus 4\,I_1\,, & \\ \mathrm{NS}(\mathbf{X}) & = & H \oplus E_8 \oplus E_8\,, & \text{signature: } \overbrace{(1,17)}^{\rho=18}\,, \\ \mathrm{T}_{\mathbf{X}} & = & H^2\,, & \text{signature: } (2,2)\,. \end{array}$$

$c \neq 0$: One Wilson line. Gauge group $G = E_8 \times E_7$.

$$\begin{array}{lcll} \text{sing. fibers of } \bar{\mathbf{X}} & : & II^* \oplus III^* \oplus 5\,I_1\,, & \\ \mathrm{NS}(\mathbf{X}) & = & H \oplus E_8 \oplus E_7\,, & \text{signature: } \overbrace{(1,16)}^{\rho=17}\,, \\ \mathrm{T}_{\mathbf{X}} & = & H^2 \oplus \langle -2 \rangle\,, & \text{signature: } (2,3)\,. \end{array}$$

Shioda-Inose correspondence

Def.: A K3 surface **X** admits a Shioda-Inose structure if there is an Abelian surface **A** and rational maps of degree 2 in diagram below such that $\mathrm{T}_{\mathbf{X}} \cong \mathrm{T}_{\mathbf{A}} =: \mathrm{T}$.

$$X \xrightarrow{\;2\;} Km(A) \xleftarrow{\;2\;} A$$

Morrison '84: For $\rho = 19, 18, 17$, every algebraic K3 surface **X** *has a Shioda-Inose structure.*

Dolgashev '96: Coarse moduli spaces $\mathcal{M}$ + global Torelli maps exist for pseudo-ample N-polarized K3's/principally polarized Abelian surfaces:

$$\mathrm{per} : \mathcal{M} \xrightarrow{\cong} O(\mathrm{T})\backslash\left\{\Omega \in \mathbb{P}^1(\mathrm{T}\otimes\mathbb{C})\,\middle|\,(\Omega,\Omega)=0,\ (\Omega,\bar{\Omega})>0\right\}$$

Resulting picture for heterotic/F-theory duality

(based on Clingher-Doran '07, '10; A. Kumar '08; M.-Morrison '11)

c/z	F-theory moduli for $\bar{\mathbf{X}} \to \mathbb{CP}^1$	heterotic moduli for $\mathbf{A}$
$= 0$	$\mathrm{T}_{\mathbf{X}} = \mathrm{T}_{\mathbf{A}} = H^2$	
	$\mathrm{NS}(\mathbf{X}) = H \oplus E_8 \oplus E_8$ $a \simeq E_4(\tau)\, E_4(\rho)$ $b \simeq E_6(\tau)\, E_6(\rho)$ $d \simeq \eta^{24}(\tau)\, \eta^{24}(\rho)$	$\mathbf{A} = E_\tau \times E_\rho$ $E_\tau : \quad j(\tau) = E_4^3(\tau)/\eta^{24}(\tau)$ $E_\rho : \quad j(\rho) = E_4^3(\rho)/\eta^{24}(\rho)$
	$(\tau, \rho) \in \Gamma \backslash \mathrm{SO}(2,2)/\big(\mathrm{SO}(2) \times \mathrm{SO}(2)\big) = \Gamma \backslash \mathbb{H} \times \mathbb{H}$	
$\neq 0$	$\mathrm{T}_{\mathbf{X}} = \mathrm{T}_{\mathbf{A}} = H^2 \oplus \langle -2 \rangle$	
	$\mathrm{NS}(\mathbf{X}) = H \oplus E_8 \oplus E_7$ $a \simeq \psi_4(\underline{\tau})\,, \quad b \simeq \psi_6(\underline{\tau})\,,$ $c \simeq \chi_{10}(\underline{\tau})\,, \quad d \simeq \chi_{12}(\underline{\tau})\,.$	$\mathbf{A} = \mathrm{Jac}\, C_{\underline{\tau}}$ $[I_2 : I_4 : I_6 : I_{10}] \in \mathbb{WP}^3_{(2,4,6,10)}$ $I_2 \simeq \frac{\chi_{12}}{\chi_{10}}, \quad I_4 \simeq \psi_4,$ $I_6 \simeq \psi_6 + \frac{\psi_4\, \chi_{12}}{\chi_{10}}, \quad I_{10} \simeq \chi_{12}$
	$\underline{\tau} = \begin{pmatrix} \tau & z \\ z & \rho \end{pmatrix} \in \Gamma \backslash \mathrm{SO}(2,3)/\big(\mathrm{SO}(2) \times \mathrm{SO}(3)\big) = \mathrm{Sp}(4, \mathbb{Z}) \backslash \mathbb{H}_2$	